Money and risk in a DSGE framework: A Bayesian application to the Eurozone

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- New Keynesian models
- Selected papers

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- Non-separable utility function
- Solving the model
- Log-linearized model

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- Calibration
- Estimation
- Simulations

Conclusion
The question of money

- In the current New Keynesian literature, the role of monetary aggregates is generally neglected.
- The main economic variables of this kind of models are: the output gap, inflation and the interest rate.
- Yet it’s hard to imagine money completely “passive” to the rest of the system!
Brunner and Meltzer

- As individuals re-allocate their portfolio of assets, the behavior of real money balances induces relative **price adjustments** on financial and real assets. In the process, **aggregate demand** changes, and thus output.

- Thus, by affecting aggregate demand, real money balances become part of the transmission mechanism.

- The interest rate alone is thus not sufficient to explain the impact of monetary policy and the role played by credit and financial markets.

- This monetarist transmission process may also imply a specific role to real money balances when dealing with **risk aversion**.
Money and new Keynesian models

- Most of studies about New Keynesian models ignore money because of separable utilities, such as following:

$$E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} \left( \frac{M_{t+i}}{P_{t+i}} \right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right]$$

- Solving this problem makes money completely recursive to the rest of the system of equations. Yet, real money holdings could affect household’s consumption.

- In other words, real money balances are supposed to affect the marginal utility of consumption, i.e. we have to assume **non-separable utility** between consumption and real money balances.
Selected papers


New Keynesian framework

The model consists of households that supply labor, purchase goods for consumption, and hold money and bonds, and firms that hire labor and produce and sell differentiated products in monopolistically competitive goods markets (Dixit and Stiglitz, 1977).

Economic agents of 3 types:

- Households maximize the expected present value of utility.
- Firms maximize profits.
- Central bank controls the nominal rate of interest.
Non-separable money in the utility

- Preferences of the representative household are defined over a composite consumption good $C_t$, real money balances $\frac{M_t}{P_t}$, and leisure $1 - N_t$, where $N_t$ is the time devoted to market employment.

- CES utility function:

$$U_t = e^{\varepsilon P} \left( \frac{1}{1-\sigma} \left( (1 - b) C_t^{1-v} + b e^{\varepsilon M} \left( \frac{M_t}{P_t} \right)^{1-v} \right)^{\frac{1-\sigma}{1-v}} - \frac{\chi N_t^{1+\eta}}{1+\eta} \right)$$

- Budget constraint:

$$P_t C_t + Q_t B_t + M_t \leq B_{t-1} + M_{t-1} + W_t N_t$$

- Production function:

$$Y_t = A_t N_t^{1-\alpha}$$
Solving the model

- By using Lagrangian method in order to optimize the utility function with respect to the budget constraint, we obtain three first-order optimal conditions.
- We log-linearize around the steady state these conditions.
- We add an ad-hoc Taylor type rule equation to close our model.
- Finally, we have 6 equations of 6 unknown variables for our economy: output gap ($\hat{y}_t$) and its flexible-price counterpart ($\hat{y}^f_t$), inflation rate ($\hat{\tau}_t$), real money balances ($\hat{mp}_t$) and its flexible-price counterpart ($\hat{mp}^f_t$) and nominal interest rate ($\hat{i}_t$).
- Structural shocks are assumed to follow a first-order autoregressive process with an $i.i.d.$-normal error term such as $\varepsilon^k_t = \rho_k \varepsilon^k_{t-1} + \omega_{k,t}$ where $\varepsilon_{k,t} \sim N(0; \sigma_k)$ for $k = \{P, M, i, a\}$.
Log-linearized model (around the steady state)

\[ \hat{y}_t^f = v_a^y \varepsilon_t + v_m^y \hat{m}p_t^f - v_c^y + v_{sm}^y \varepsilon_t^M \]  

(1)

\[ \hat{m}p_t^f = v_{y+1}^m E_t [\hat{y}_{t+1}^f] + v_y^m \hat{y}_t^f + \frac{1}{v} \varepsilon_t^M \]  

(2)

\[ \hat{\pi}_t = \beta E_t [\hat{\pi}_{t+1}] + \kappa_x (\hat{y}_t - \hat{y}_t^f) + \kappa_m (\hat{m}p_t - \hat{m}p_t^f) \]  

(3)

\[ \hat{y}_t = E_t [\hat{y}_{t+1}] - \kappa_r (\hat{i}_t - E_t [\hat{\pi}_{t+1}]) + \kappa_m E_t [\Delta \hat{m}p_{t+1}] \] 

\[ + \kappa_{sp} E_t [\Delta \varepsilon^p_{t+1}] + \kappa_{sm} E_t [\Delta \varepsilon^M_{t+1}] \]  

(4)

\[ \hat{m}p_t = \hat{y}_t - \kappa_i \hat{i}_t + \frac{1}{v} \varepsilon_t^M \]  

(5)

\[ \hat{i}_t = (1 - \lambda_i) \left( \lambda_\pi \hat{\pi}_t + \lambda_x (\hat{y}_t - \hat{y}_t^f) \right) + \lambda_i \hat{i}_{t-1} + \varepsilon_t^i \]  

(6)
Micro-funded model

\[ \nu_a^y = \frac{1 + \eta}{(v - (v - \sigma)a_1)(1 - \alpha) + \eta + \alpha} \]

\[ \nu_m^y = \frac{1 + \eta}{(v - (v - \sigma)a_1)(1 - \alpha) + \eta + \alpha} \]

\[ \nu_c^y = \frac{1 - \alpha \log \left( \frac{\nu}{\nu - (v - \sigma)a_1} \right)}{(v - (v - \sigma)a_1)(1 - \alpha) + \eta + \alpha} \]

\[ \nu_{ym}^{y+1} = -\frac{a_2}{v} (v - (v - \sigma) a_1) \]

\[ \nu_{ym}^y = 1 + \frac{a_2}{v} (v - (v - \sigma) a_1) \]

\[ \kappa_m = (1 - a_1)(\sigma - v) \frac{(1 - \alpha)(1 - \beta\theta)(1 - \theta)}{\theta(1 + a_1 + \theta)} \]

\[ \kappa_x = (v - (v - \sigma) a_1 + \frac{\eta + \alpha}{1 - \alpha}) \frac{(1 - \alpha)(1 - \beta\theta)(1 - \theta)}{\theta(1 + a_1 + \theta)} \]

\[ \kappa_r = \frac{1}{v - a_1(v - \sigma)} \]

\[ \kappa_{mp} = \frac{(\sigma - v)(1 - a_1)}{v - a_1(v - \sigma)} \]

\[ \kappa_{sp} = -\frac{1}{v - a_1(v - \sigma)} \]

\[ \kappa_{sm} = -\frac{(1 - a_1)(v - \sigma)}{v - a_1(v - \sigma)(1 - v)} \]

\[ \kappa_i = \frac{a_2}{v} \]

\[ a_1 = \frac{1}{1 + (b/(1-b))^{1/v} (1 - \beta)^{(v-1)/v}} \]

\[ a_2 = \frac{1}{\exp(1/\beta) - 1} \]
Methodology

- As in Smets and Wouters (2003), An and Schorfheide (2007) and Barthélemy, Clerc and Marx (2011), we apply **Bayesian techniques** to estimate our DSGE model.
- We use **Eurozone data** like Andrès et al. (2006) and Barthélemy, Clerc and Marx (2011) from the Euro Area Wide Model database (AWM) of Fagan, Henry and Mestre (2001).
- We use the *M3* monetary aggregate from the OECD database, which is the broadest monetary aggregate.
- To make output and real money balances stationary, we use first log differences, as in Adolfson and *al.* (2008).
Introduction

The model

Methodology

Calibration

Estimation

Simulation

Results

Comments and conclusion

Data

- $\hat{\pi}_t$ is the log-linearized inflation rate measured as the yearly log difference of GDP Deflator from one quarter to the same quarter of the previous year;
- $\hat{y}_t$ is the log-linearized output measured as the yearly log difference of GDP from one quarter to the same quarter of the previous year;
- $i_t$ is the short-term (3-month) nominal interest rate.
- $\hat{mp}_t$ is the log-linearized real money balances measured as the yearly log difference of real money from one quarter to the same quarter of the previous year, where real money is measured as the log difference between the money stock and the GDP Deflator.
- $\hat{y}_t^f$, the flexible-price output, and $\hat{mp}_t^f$, the flexible-price real money balances, are completely determined by structural shocks.
Calibration

- Following standard conventions, we calibrate **beta** distributions for parameters that fall between zero and one, **inverted gamma** distributions for parameters that need to be constrained to be greater than zero, and **normal** distributions in other cases.

- The calibration of $\sigma$ is inspired by Rabanal and Rubio-Ramírez (2007) and by Casares (2007), respectively of 2.5 and 1.5.

- $\sigma = 2$ corresponds to a standard risk aversion while values.

- $\sigma = 4$, twice the standard value, represents a high level of risk aversion.

- As our goal is not to estimate risk aversion but to analyze two different configurations of risk, we adopt the same priors in the two models with different risk aversion calibration.

- Detailed calibration in the paper.
Methodology

- **Sample**: 117 observations from 1980 (Q4) to 2009 (Q4) in order to avoid high volatility periods before 1980.

- **Algorithm**: Metropolis-Hastings of 10 distinct chains, each of 100000 draws (Smets and Wouters, 2007; Adolfson et al., 2007).

- **Average acceptation rate per chain for the benchmark model** ($\sigma = 2$) are included in the interval $[0.2601; 0.2661]$ and for ($\sigma = 4$) in the interval $[0.2587; 0.2658]$. 
### Bayesian estimation of structural parameters (1)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Prior Distribution</th>
<th>Prior Mean</th>
<th>Prior Std.</th>
<th>Posterior Mean for $\sigma = 2$</th>
<th>Posterior Mean for $\sigma = 4$</th>
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### Bayesian estimation of structural parameters (2)

<table>
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<th>Priors</th>
<th>Posteriors</th>
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<th>$\sigma = 4$</th>
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## Simulations with posterior means

### Unconditional variance decomposition (%)

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<th>with $\sigma = 4$</th>
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<td>$\varepsilon_t^P$</td>
<td>$\varepsilon_t^i$</td>
<td>$\varepsilon_t^M$</td>
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<td>$\hat{y}_t$</td>
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<td>26.92</td>
<td><strong>4.47</strong></td>
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<tr>
<td>$\hat{i}_t$</td>
<td>24.72</td>
<td>74.16</td>
<td>0.06</td>
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<tr>
<td>$\hat{mp}_t$</td>
<td>0.43</td>
<td>0.15</td>
<td>75.75</td>
</tr>
<tr>
<td>$\hat{y}_f^c$</td>
<td>0</td>
<td>0</td>
<td><strong>5.6</strong></td>
</tr>
<tr>
<td>$\hat{mp}_f^c$</td>
<td>0</td>
<td>0</td>
<td>74.75</td>
</tr>
</tbody>
</table>

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Conditional variance decomposition of output

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Shock decomposition of output

Under standard (above) and high (below) risk aversion ($\varepsilon_P^t$ in blue, $\varepsilon_M^t$ in yellow, $\varepsilon_i^t$ in azure and $\varepsilon_a^t$ in red).
Interpretation

- The weight of the money shock on output dynamics, $\kappa_{sm}$, and on flexible-price output, $\nu^y_{sm}$, increases with risk aversion.
- **The higher the risk aversion, the higher the role of money on output.**
- The central bank strives for financial stability in crisis periods. The smoothing parameter in the Taylor rule equation, $\lambda_i$, increases with risk aversion.
Conclusion

- Under a standard risk aversion: money plays a minor role in explaining output variability (Andrès et al., 2006; Ireland, 2004).
- **Under a higher risk aversion:** money plays a non-negligible role in explaining output and flexible-price output fluctuations.
- The explicit money variable does not appear to have a notable direct role in explaining inflation variability.
- The higher the risk aversion, the stronger the smoothing of the interest rate. This reflects probably the central bankers’ objective not to agitate markets.
- One may infer that by changing economic agents’ perception of risks, the last financial crisis may have increased the role of money in the transmission mechanisms and in output changes.
Further research

Out-of-sample forecasting errors (DSGE Forecast)

Comparison of output and inflation DSGE forecast errors
Positive bar: Non-Separable model is better
Negative bar: Baseline model is better

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Further research

Comparison between the role of money on output and the spreads between the Bubill/BTF and the Euribor