

# Central bank losses and monetary policy rules: a DSGE investigation\*

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## Abstract

Central banks' monetary policy rules being consistent with policy objectives are a fundamental of applied monetary economics. We seek to determine, first, which of the central bank's rules are most in line with the historical data for the US economy and, second, what policy rule would work best to assist the central bank in reaching its objectives via several loss function measures. We use Bayesian estimations to evaluate twelve monetary policy rules from 1955 to 2017 and over three different sub-periods. We find that when considering the central bank's loss functions, the estimates often indicate the superiority of NGDP level targeting rules, though Taylor-type rules lead to nearly identical implications. However, the results suggest that various central bank empirical rules, be they NGDP or Taylor type, are more appropriate to achieve the central bank's objectives for each type of period (stable, crisis, recovery).

*Keywords:* Monetary policy, Monetary rule, Central bank loss.

*JEL Classification:* E52, E58, E32.

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# 1 Introduction

Monetary economists generally contend that central bankers should follow policy rules rather than use their own discretion when devising monetary policy. Debates held during the 1970s and 1980s suggested nominal income targeting concepts, even if they were not always presented as such.<sup>1</sup> The consensus on Taylor (1993) rules increased during the last two decades.<sup>2</sup> However, criticism of such monetary policy rules also increased,<sup>3</sup> especially during and after the Global Financial Crisis<sup>4</sup> (GFC/ZLB), arguing that nominal income targeting could be a better way to achieve the central banks' objectives.

An interesting way to compare and evaluate different monetary policy proposals and rules is to introduce them within the framework of a macroeconomic Dynamic Stochastic General Equilibrium (DSGE) model. Because the dynamics are so important and difficult to work through intuitively, such empirical models can provide invaluable clarification of the matter (Taylor, 2013).

Our aim in this paper is to use the Smets and Wouters (2007) framework, the well-known baseline DSGE model fitted for the US, to evaluate different monetary policy rules and their consequences in terms of current and forecasted central bank objectives.

These objectives may differ for various reasons, hence the need to analyze several hypotheses regarding the current and forecasted preferences of the central banker. Such an approach implies an analysis of the impact of policies on central bank loss functions (Taylor and Wieland, 2012; Walsh, 2015).

Two main research strategies may be used to deal with these issues: a rather common one using optimal monetary policy theory and an empirical one based on historical economic dynamics. The latter appears better suited to capture the real economic behavior of a central bank. Indeed, commitment, discretionary or optimal monetary rules imply theoretically optimal behavior but do not necessarily represent the actual behavior of central bankers. In addition, this theoretical framework does not permit the analysis of empirical central bank losses over time.

Since central banks do not necessarily behave optimally, our empirical exercise offers a more realistic framework regarding central bank behavior

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<sup>1</sup>See Friedman (1971), Meade (1978), and McCallum (1973, 1987).

<sup>2</sup>See Bernanke and Mishkin (1997), Svensson (1999), and Taylor (1999).

<sup>3</sup>See for instance Hall and Mankiw (1994), Frankel and Chinn (1995), McCallum and Nelson (1999), and Rudebusch (2002a).

<sup>4</sup>Hendrickson (2012), Woodford (2012), Frankel (2014), Sumner (2014, 2015), Belongia and Ireland (2015), and McCallum (2015) for example.

than the theoretical one. Finally, the implications of ad-hoc monetary policy rules are rarely analyzed in terms of various central bank losses, be they current or forecasted, especially during different sample periods, which we consider in our analysis. Our approach allows for such an analysis through a medium-scale DSGE model.

The monetary policy rules we examine are of three types: Taylor-type rules, nominal income *growth* rules, and nominal income *level* rules. There are four Taylor-type rules, following (1) a structure *à la* Smets and Wouters (2007), where the nominal interest rate responds to an inflation gap, an output gap and output gap growth; (2) a structure *à la* Taylor (1993), where the nominal interest rate responds to an inflation gap and an output gap; (3) a structure *à la* Galí (2015), where the nominal interest rate responds to an inflation gap, an output gap and a natural interest rate defined as the interest rate in the flexible-price economy; and (4) a structure *à la* Garín et al. (2016), where the nominal interest rate responds to an inflation gap and to output growth. There are also four nominal GDP (NGDP) growth rules that replace the core functions of the Taylor-type rules with an NGDP growth targeting function. Finally, our last four rules replace the core functions of the Taylor-type rules with an NGDP level rule.

We apply Bayesian techniques to estimate our twelve DSGE models (each type is composed of 4 structures) using US data. Note that this approach goes further than the literature generally does. First, we consider a large set of monetary policy rules. Second, our models are studied over different time periods. Third, we add the analysis of central bank losses, current and forecasted, over these models and periods. Fourth, the model structure we use (Smets and Wouters, 2007) is a sophisticated medium-scale model.<sup>5</sup> We believe that our analysis and estimates enrich the literature in an informative and innovative way.

Specifically, we estimate all of the parameters over several sample periods: the overall available sample (1955-2017) and three sub-samples, each with different economic environments and monetary policy styles, running from 1955 to 1985, from 1985 to 2007 and from 2007 to 2017.

Monetary policy during the GFC/ZLB period can hardly be described by a monetary policy rule in which the monetary policy shock is assumed to be normally distributed. To overcome this statistical problem, while taking into consideration most of such unconventional monetary policies (credit easing, quantitative easing, and forward guidance), we use the shadow rate<sup>6</sup> (Kim and Singleton, 2012; Krippner, 2013). The shadow rate is a version of the

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<sup>5</sup>Models like this have been extensively used for policy analysis at various central banks.

<sup>6</sup>Wu and Xia (2016) devised a shadow Fed funds rate that can be negative, reflecting the Fed's unconventional policies. When quantitative easing or forward guidance is pursued,

federal funds rate that can take negative values; it is also consistent with a term structure of interest rates. Thus, it allows for meaningful monetary policy analysis and interpretation during low interest rate regimes, without ignoring data from high interest rate periods.

From the estimations and simulations of our models, we analyze, among other factors, the monetary policy rules' parameters, in-sample fits (which monetary rule is most in line with historical data) and the central bank's loss functions, current or forecasted. Estimated parameters, estimated shocks, impulse response functions, and variance decompositions are presented in the online appendix.

We find that when considering the central bank's loss functions, the estimates often indicate the superiority of NGDP level rules, although Taylor-type rules have nearly identical implications. However, this being given, the results suggest that historical fitting and the central bank's objectives cannot be achieved by one single rule over all time frames. For each type of period (more or less stable, crisis, recovery), a specific rule performs better than others. Policy institutions, which base their forecasts and policy recommendations on such models and rules, should refresh their estimates regularly because the parameter estimates of the rule vary over time.

The remainder of the paper is organized as follows. Section 2 describes the theoretical setup. Section 3 describes the empirical methodology. Monetary rule parameters estimates as well as in-sample fit results and analysis are presented in Section 4. Central bank loss measures are presented in Section 5. Our results are interpreted in Section 6. Section 7 draws some policy implications, and Section 8 concludes the paper. The online appendix presents additional empirical results.

## 2 The models

The [Smets and Wouters \(2007\)](#) model is the core model used in this paper. However, in their article and other working paper versions, those authors do not describe a flexible-price economy. We perform this work in the detailed description of the log-linearized sticky- and flexible-price economies in our online appendix.

This (generic) model, also detailed in the online appendix, needs to be completed by adding an ad hoc monetary policy reaction function (Table 1).

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the Fed's current rate is zero (ZLB), while the shadow rate changes. When rates are above the ZLB, the shadow rate is identical to the Fed funds rate. Once the ZLB is reached, the [Wu and Xia \(2016\)](#) rate uses a Gaussian affine term structure model to generate an effective rate.

Despite their different formulations, all of these functions include a smoothing process that captures the degree of rule-specific smoothing.

### Taylor-type rules

- Model **1** is the original [Smets and Wouters \(2007\)](#) monetary policy rule, which gradually responds to deviations of inflation ( $\pi_t$ ) from an inflation objective (normalized to zero), the output gap, defined as the difference between sticky-price ( $y_t$ ) and flexible-price ( $y_t^p$ ) outputs (see the online appendix), and deviations of the output gap from the previous period ( $\Delta y_t - \Delta y_t^p$ ).
- Model **2** is based on the [Taylor \(1993\)](#) monetary policy rule, which gradually responds to deviations of inflation from an inflation objective (normalized to zero) and of the output gap, as previously defined.<sup>7</sup>
- Model **3** is the [Galí \(2015\)](#) monetary policy rule, which gradually responds to the natural interest rate ( $r_t^*$ ), as defined in [Galí \(2015\)](#), deviations of inflation from an inflation objective (normalized to zero) and of the output gap, as previously defined.
- Model **4** gradually responds to the deviations of inflation from an inflation objective (normalized to zero) and output growth ([Iacoviello and Neri, 2010](#); [Garín et al., 2016](#)). It assumes that the natural output ( $y_t^p$ ), as well as the natural interest rate, are not observable in real time.

### Nominal GDP growth rules

- Model **5** is the Adapted NGDP Growth targeting monetary policy rule, which gradually responds to deviations of nominal output growth ( $\pi_t + \Delta y_t$ ) from an objective, as in [McCallum and Nelson \(1999\)](#), and deviations of the output gap from the previous period (output gap growth, as in model 1).
- Model **6** is the NGDP Growth targeting monetary policy rule, which gradually responds to deviations of nominal output growth from its flexible-price counterpart (FPC).

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<sup>7</sup>In the original Taylor rule, the natural interest rate is constant ([Taylor, 1993](#)). Log-linearization around the steady state eliminates this (constant) natural interest rate. Note that rule 1 also ([Smets and Wouters, 2007](#)) does not include the natural interest rate.

- Model **7** is the NGDP Growth targeting monetary policy rule including a natural interest rate (NIR) component, where the policy gradually responds to the NIR, as in [Rudebusch \(2002a\)](#), and deviations of nominal output growth from its flexible-price counterpart.
- Model **8** is the NGDP Growth targeting monetary policy rule where the policy gradually responds to the deviations of nominal output growth.

### Nominal GDP level rules

- Model **9** is the Adapted NGDP Level targeting monetary policy rule, which gradually responds to nominal output level ( $p_t + y_t$ ) deviations from its flexible-price counterpart,<sup>8</sup> as suggested by [McCallum \(2015\)](#), and deviations of the output gap from the previous period (output gap growth, as in model 1).
- Model **10** is the NGDP Level targeting monetary policy rule, which gradually responds to nominal output level deviations from its flexible-price counterpart (FPC).
- Model **11** is the NGDP Level targeting monetary policy rule including an NIR component, where the policy gradually responds to the NIR and to deviations of the nominal output level from its flexible-price counterpart.
- Model **12** is the NGDP Level targeting monetary policy rule where the policy gradually responds to the nominal output level.

As indicated above, there are three categories of rules. The first four (1 to 4) are of the Taylor-type. Rules 5 to 8 are nominal GDP rules targeting nominal GDP growth. Rules 9 to 12 target the level of nominal GDP.

Rules 5 and 9 include output gap growth, as in rule 1 ([Smets and Wouters, 2003, 2007](#)). Rules 7 and 11 include the natural interest rate, as in rule 3 ([Galí, 2015](#)). Including these variables allows us to compare the various rules with their standard versions as presented by the above-cited authors.

These three categories of rules represent the main policy rules in the contemporary literature.

As these rules are all *ad hoc*, they do not require changes in the specification of the core model. The unique differentiating feature of the twelve

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<sup>8</sup>The level of nominal output is  $p_t + y_t$ , where prices  $p_t$  are deducted from the definition of inflation  $\pi_t = p_t - p_{t-1}$ .

Models	Sources	Monetary policy rules
1	Smets and Wouters (2007)	$r_t = \rho r_{t-1} + (1 - \rho) [r_\pi \pi_t + r_y (y_t - y_t^p)] + r_{\Delta y} (\Delta y_t - \Delta y_t^p) + \varepsilon_t^r$
2	Taylor (1993)	$r_t = \rho r_{t-1} + (1 - \rho) [r_\pi \pi_t + r_y (y_t - y_t^p)] + \varepsilon_t^r$
3	Galí (2015)	$r_t = \rho r_{t-1} + (1 - \rho) [r_t^* + r_\pi \pi_t + r_y (y_t - y_t^p)] + \varepsilon_t^r$
4	Garín et al. (2016)	$r_t = \rho r_{t-1} + (1 - \rho) [r_\pi \pi_t + r_y \Delta y_t] + \varepsilon_t^r$
5	Adapted NGDP Growth Targeting	$r_t = \rho r_{t-1} + (1 - \rho) [r_n (\pi_t + \Delta y_t - \Delta y_t^p)] + r_{\Delta y} (\Delta y_t - \Delta y_t^p) + \varepsilon_t^r$
6	NGDP Growth + FPC Targeting	$r_t = \rho r_{t-1} + (1 - \rho) [r_n (\pi_t + \Delta y_t - \Delta y_t^p)] + \varepsilon_t^r$
7	NGDP Growth + NIR Targeting	$r_t = \rho r_{t-1} + (1 - \rho) [r_t^* + r_n (\pi_t + \Delta y_t - \Delta y_t^p)] + \varepsilon_t^r$
8	NGDP Growth Targeting	$r_t = \rho r_{t-1} + (1 - \rho) [r_n (\pi_t + \Delta y_t)] + \varepsilon_t^r$
9	Adapted NGDP Level Targeting	$r_t = \rho r_{t-1} + (1 - \rho) [r_n (p_t + y_t - y_t^p)] + r_{\Delta y} (\Delta y_t - \Delta y_t^p) + \varepsilon_t^r$
10	NGDP Level + FPC Targeting	$r_t = \rho r_{t-1} + (1 - \rho) [r_n (p_t + y_t - y_t^p)] + \varepsilon_t^r$
11	NGDP Level + NIR Targeting	$r_t = \rho r_{t-1} + (1 - \rho) [r_t^* + r_n (p_t + y_t - y_t^p)] + \varepsilon_t^r$
12	NGDP Level Targeting	$r_t = \rho r_{t-1} + (1 - \rho) [r_n (p_t + y_t)] + \varepsilon_t^r$

NIR and FPC stand for the natural interest rate ( $r_t^*$ ) and the flexible-price counterpart *à la* Galí (2015), respectively.

Table 1: Summary of monetary policy rules used in this study



models therefore comes from their respective monetary policy rule. Concerning NGDP Level targeting rules (models 9 to 12), we add to the core model and the monetary policy rule the definition of prices, derived from (in log form)  $\pi_t = p_t - p_{t-1}$ , where  $p_t$  represents the log-price index at time  $t$ .

In addition, the inflation rate in the flexible-price economy at time  $t$  is  $\pi_t^p = 0$ , as in Smets and Wouters (2007). Then, the flexible-price nominal income is only defined by  $\Delta y_t^p$  (growth) or  $y_t^p$  (level). These assumptions are used in rules 5 to 7 (NGDP Growth rules) and 9 to 11 (NGDP Level rules) in Table 1.

## 3 Methodology

### 3.1 Data

The models, with various monetary policy rules, are estimated between 1955 and 2017 and over three different periods within this time interval: from 1955Q1 to 1985Q1, a period when the economy was rather unstable and featured ups and downs and monetary policy could be characterized as discretionary; from 1985Q1 to 2007Q1, the *Great Moderation era* (GM), when the economy was rather stable and monetary policy more predictable; and from 2007Q1 to 2017Q1, the *GFC/ZLB era*, the crisis and recovery period when monetary policy followed an unusual ZLB track.

During our first sub-sample (1955-1985), monetary policy was rather discretionary and severely criticized in the literature (Friedman, 1982). Since the 1980s, the predictability and stability of monetary policy has improved, with many researchers currently recommending rule-based rather than discretionary monetary policy decisions (Kydland and Prescott, 1977; Taylor, 1986, 1987; Friedman, 1982; Taylor, 1993). Notice that monetary policies occurring during our first sub-sample (1955-1985) were often modeled by a rule in the literature (Smets and Wouters, 2007; Nikolsko-Rzhevskyy and Papell, 2012; Nikolsko-Rzhevskyy et al., 2014).

Our second sub-sample (1985-2007) is inspired by Clarida (2010), describing the period 1985-2007 as the GM. Although our second sub-sample is in line with the literature (Clarida, 2010; Meltzer, 2012; Taylor, 2012; Nikolsko-Rzhevskyy et al., 2014), we extend it until 2007, to define a sub-sample with a relatively stable economy (despite the dot-com crisis beginning in the 2000s) that can be compared with the crisis period starting in 2007.

Our third sub-sample (2007-2017) is well documented in the crisis and recovery period literature (Gorton, 2009; Cúrdia and Woodford, 2011; Benchi-mol and Fourçans, 2017).

The series are quarterly, and data transformations, data sources<sup>9</sup> and measurement equations<sup>10</sup> are exactly the same as in [Smets and Wouters \(2007\)](#).

We estimate our models over the third sub-sample (2007-2017) by using the shadow rate<sup>11</sup> data for the US pursuant to [Wu and Xia \(2016\)](#).

### 3.2 Calibration

To maintain consistency across models for comparison purposes, we specify and calibrate prior distributions for all model parameters as in [Smets and Wouters \(2007\)](#). A detailed description of these parameters, and their calibrations, is provided in the online appendix.

Except for NGDP targeting rules, monetary policy rule parameters in [Table 2](#) have the same calibration as in [Smets and Wouters \(2007\)](#).

	<b>Law</b>	<b>Mean</b>	<b>Std.</b>
$\rho$	Beta	0.75	0.10
$r_\pi$	Normal	1.50	0.25
$r_y$	Normal	0.125	0.05
$r_{\Delta y}$	Normal	0.125	0.05
$r_n$	Normal	$1.5^{(*)}/0.5^{(**)}$	0.25

Table 2: Prior distribution of monetary policy rule parameters. (\*) stands for NGDP growth targeting (rules 5 to 8). (\*\*) stands for NGDP level targeting (rules 9 to 12).

Of course,  $r_{\Delta y}$  equals zero in models 2 to 4, 6 to 8, and 10 to 12.  $r_\pi$  and  $r_y$  are not used in models 5 to 12, and  $r_n$  is not used in models 1 to 4.

As explained in [Rudebusch \(2002a\)](#),  $r_n$  is higher than one for NGDP growth targeting rules and positive and smaller than one for NGDP level targeting rules.

### 3.3 Estimation

As in [Smets and Wouters \(2007\)](#), we apply Bayesian techniques to estimate our DSGE models with different specifications of monetary policy rules. We

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<sup>9</sup>Detailed data sources, measurement equations and data transformations are available in the online appendix.

<sup>10</sup>Measurement equations are presented in the online appendix.

<sup>11</sup>From December 16, 2008, to December 15, 2015, the effective federal funds rate was in the 0 to 1/4 percent range. In this zero lower bound environment, shadow rate models are used ([Kim and Singleton, 2012](#); [Krippner, 2013](#)).

estimate all the parameters presented above over the four different periods defined in Section 3.1.

To achieve draw acceptance rates between 20% and 40%, we calibrate the tuning parameter on the covariance matrix for each model and each period. Our results, for each model and each period, are based on the standard Monte Carlo Markov Chain (MCMC) algorithm with 6 000 000 draws of 2 parallel chains (where 3 000 000 draws are used for burn-in).

To avoid undue complexity, we do not present all the estimates. We prefer to concentrate on the analysis of the parameters of the different monetary rules. All the estimation results<sup>12</sup> are available in the online appendix. These results confirm well-identified parameters and shock estimates in line with the literature. The comparison of prior and posterior distributions for approximately 35 estimated parameters, for all 48 estimations (12 rules for each 4 periods), does not highlight any identification issues.

## 4 Monetary rule parameters and in-sample fit

Parameter estimates are detailed in the online appendix with all impulse response functions and variance decompositions. To draw policy conclusions from our models, we assess monetary policy rule parameters (estimated values) in Section 4.1 and the models' in-sample fit in Section 4.2.

### 4.1 Monetary rule parameters

Fig. 1 presents the estimates of the smoothing parameter ( $\rho$ ), the inflation coefficient ( $r_\pi$ ), the output gap coefficient ( $r_y$ ), the output gap growth coefficient ( $r_{\Delta y}$ ) and the nominal income coefficient ( $r_n$ ).

As Fig. 1 shows, the smoothing parameter is in line with the literature (Justiniano and Preston, 2010), at approximately 0.8, and rather stable over time, although it appears somewhat smaller for rules 9, 10 and 12, a result in accordance with Rudebusch (2002a,b).

The inflation coefficient (for rules 1 to 4) remains between 1.5 and 2, also in line with the literature (Smets and Wouters, 2007; Adolfson et al., 2011). Note that it is somewhat smaller during the GFC/ZLB, suggesting less reaction by the Fed to inflation developments than during more stable periods, notably than during the GM, from 1985 to 2007.

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<sup>12</sup>Estimated parameters (mean), estimated standard errors (std), highest posterior density intervals (HPDi) and estimated shocks are presented in the online appendix. Other detailed results are available upon request.

The value of the coefficient of the output gap varies across the periods. It appears to be higher during the GFC/ZLB period (it remains between 0.15 and 0.20) than between 1955 and 1985 (its value ranges from 0.10 to 0.15, except for rule 4). This difference is not as significant when we compare the crisis period with the 1985-2007 period (except for rule 3, to some extent).

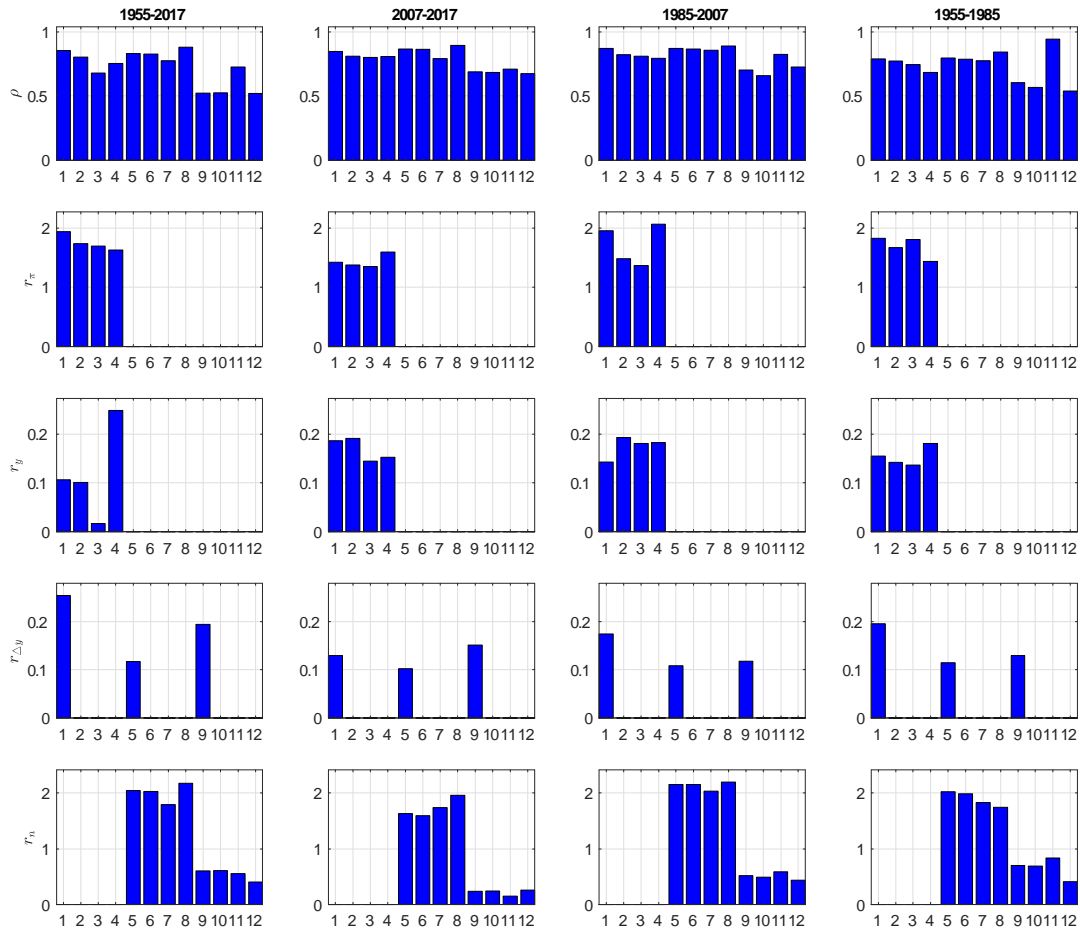


Figure 1: Monetary policy rule parameter values for each model (1 to 12).

These estimates of the Taylor-type rules (rules 1 to 4) imply a Fed that placed greater emphasis (on the margin) on the output gap during the crisis than during the previous, stabler period.

The output gap growth coefficient varies somewhat across periods and rules (between 0.10 and 0.23). At least for rule 9, this coefficient appears to be somewhat higher during the GFC/ZLB than during the GM, implying a

larger reaction to output growth during the crisis than during the previous, stabler period. For rule 1, this coefficient is the highest during the sub-period 1955-1985, yet with rule 5 it becomes the smallest, notably during the GFC/ZLB.

The nominal income coefficient associated with the NGDP rules is higher for the growth rules than the level rules, over all periods, a result that echoes Rudebusch (2002a). For the growth and level rules, this coefficient is lower during the GFC/ZLB than otherwise, especially during the GM. The coefficient for the NGDP level rules changes (with time and rule) but is lower during the GFC/ZLB period than during the other periods.

## 4.2 In-sample fit

Which monetary rule best fits the historical data is significant for understanding and analyzing the behavior of a central bank and drawing implications about policy debates. It does not mean that the central banker always followed monetary rules, but at least implicitly, he (generally) behaved “as if” he followed some kind of rule. Unveiling such rules, which may vary with the state of the economy, may clarify the background of monetary policy over time.

Furthermore, assessing in-sample fit is important to determine whether historical data (sample) are more or less in line with data generated by the estimated model. Table 3 shows the Laplace approximation<sup>13</sup> around the posterior mode (based on a normal distribution), i.e., log marginal densities, for each model and for each sample.

Sample	Rule											
	1	2	3	4	5	6	7	8	9	10	11	12
1955-2017	-1491	-1515	-1512	-1510	-1464	-1481	-1488	-1514	-1563	-1602	-1556	-1548
2007-2017	-269	-270	-285	-285	-307	-308	-283	-302	-258	-262	-278	-254
1985-2007	-386	-428	-408	-406	-406	-404	-396	-410	-393	-395	-405	-383
1955-1985	-817	-824	-835	-840	-840	-855	-837	-842	-844	-846	-863	-853

Table 3: Log marginal data densities for each rule and each period (Laplace approximation). Best values for each period in gray.

Table 3 suggests that the last NGDP rule in levels (rule 12), the pure NGDP level targeting without flexible-price output, best fits the historical data during the GFC/ZLB, yet rule 9 comes close. Rule 12 also performs best during the GM period, while rule 1, the Smets and Wouters (2007) rule,

<sup>13</sup>The Geweke (1999) mean harmonic estimator provides a similar ranking of models.

comes close. This result suggests that the Fed may have changed strategy during the GM compared to what it did before 1985. It may have switched from a Taylor type framework to an NGDP Level targeting framework, and then maintained, and even reinforced, this targeting type once the federal funds rate hit the zero lower bound.

Finally, rule 1 dominates the other rules over the period 1955-1985, whereas rule 5 ranks first over the whole sample.

For each period, a different monetary policy rule best fits the historical data, except for rule 12 that places first twice. Note that standard Taylor-type rules (rules 2 to 4) and NGDP growth targeting rules (rules 5 to 8) are generally inferior to the other rules in explaining historical data, at least over the various sub-periods.

However, this result does not imply that models with lower log marginal data densities should be discarded. Whatever the log marginal data density function, it may be argued that each model is designed to capture only certain characteristics of the data. Whether the marginal likelihood is a good measure to evaluate how well the model accounts for particular aspects of the data is an open question (Koop, 2003; Fernández-Villaverde and Rubio-Ramírez, 2004; Del Negro et al., 2007; Benchimol and Fourçans, 2017).

## 5 Central bank losses

It is traditional to assume that central banks seek to minimize a loss function based on the historical variances of the variables of interest to the bank. Generally, the current values of these variables are considered. However, the decision maker could also use forecasted values to determine which monetary policy is best as far as economic dynamics are concerned, hence our decision to analyze the minimization of two types of loss functions, one based on current (and past) outcomes in Section 5.1 and the other on forecasted ones in Section 5.2.

### 5.1 Current loss function

As noted above, the preferences of the central banker are generally represented by a loss function that he seeks to minimize. This minimization process is also supposed to represent the objectives of society.

In this section, we present current loss measures based on the historical variances of the variables of interest from the central bank's perspective. These variances are estimated for each model and for each period.

Many ad hoc central bank loss functions appear in the literature (Svensson and Williams, 2009; Taylor and Wieland, 2012; Adolfson et al., 2014). Our methodology intends to summarize all standard possibilities. For various sets of weights defining these functions, we compute the ex post loss functions consistent with the estimated DSGE model. This approach is used in the literature to investigate empirical monetary policy rules (Taylor, 1979; Fair and Howrey, 1996; Taylor, 1999) and is different from the optimal monetary policy literature (Schmitt-Grohé and Uribe, 2007; Billi, 2017).

Non-separability between consumption and labor (worked hours) in the Smets and Wouters (2007) household utility function (see the online appendix) introduces labor-related variables into the inflation and output equations. By minimizing its loss function with respect to these two equations, the central bank must also consider labor-related variables, such as wages (the price of worked hours).

Our general central bank loss function,  $L_t$ , is defined in a traditional way as<sup>14</sup>

$$L_t = var(\pi_t) + \lambda_y var(y_t - y_t^p) + \lambda_r var(\Delta r_t) + \lambda_w var(w_t) \quad (2)$$

where  $var(\cdot)$  is the variance operator,  $\lambda_y$  the weight on output gap variances,  $\lambda_r$  the weight on nominal interest rate differential variance, and  $\lambda_w$  the weight on wage inflation variance. The weight on price inflation variance is normalized to unity.  $\pi_t$  is price inflation,  $y_t - y_t^p$  the output gap,  $\Delta r_t$  the nominal interest rate differential, and  $w_t$  wage inflation.<sup>15</sup>

First, in Fig. 2, we present the estimated variances of each variable (inflation, output gap, nominal interest rate differential, and wage inflation) entering the central bank loss functions.

The variances of all variables under consideration are significantly higher before 1985 and over the full sample. Even during the 2007-2017 period, these variances were lower than before 1985 and little different from those during the GM period. The fact that estimated variances over the GFC/ZLB period are comparable across the models with those of the GM period does not mean that the variances of historical data during the GFC/ZLB and GM are comparable. Indeed, the variances presented in Fig. 2 are estimated from the models while assuming that the Fed followed various rules and the

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<sup>14</sup>See Galí (2015) for further details. Another loss measure based on the squared distance of variables generated by the models can be defined as

$$L_t = \pi_t^2 + \lambda_y (y_t - y_t^p)^2 + \lambda_r (\Delta r_t)^2 + \lambda_w w_t^2 \quad (1)$$

By the definition of the variance operator, this type of formulation leads to a ranking similar to those given by Eq. 2.

<sup>15</sup>See the online appendix for further details on the variables in the models.

US economy behaved as in the [Smets and Wouters \(2007\)](#) model. The high inflation period *cum* various significant ups and downs in economic activity and interest rates explain the high values observed between 1955 and 1985.

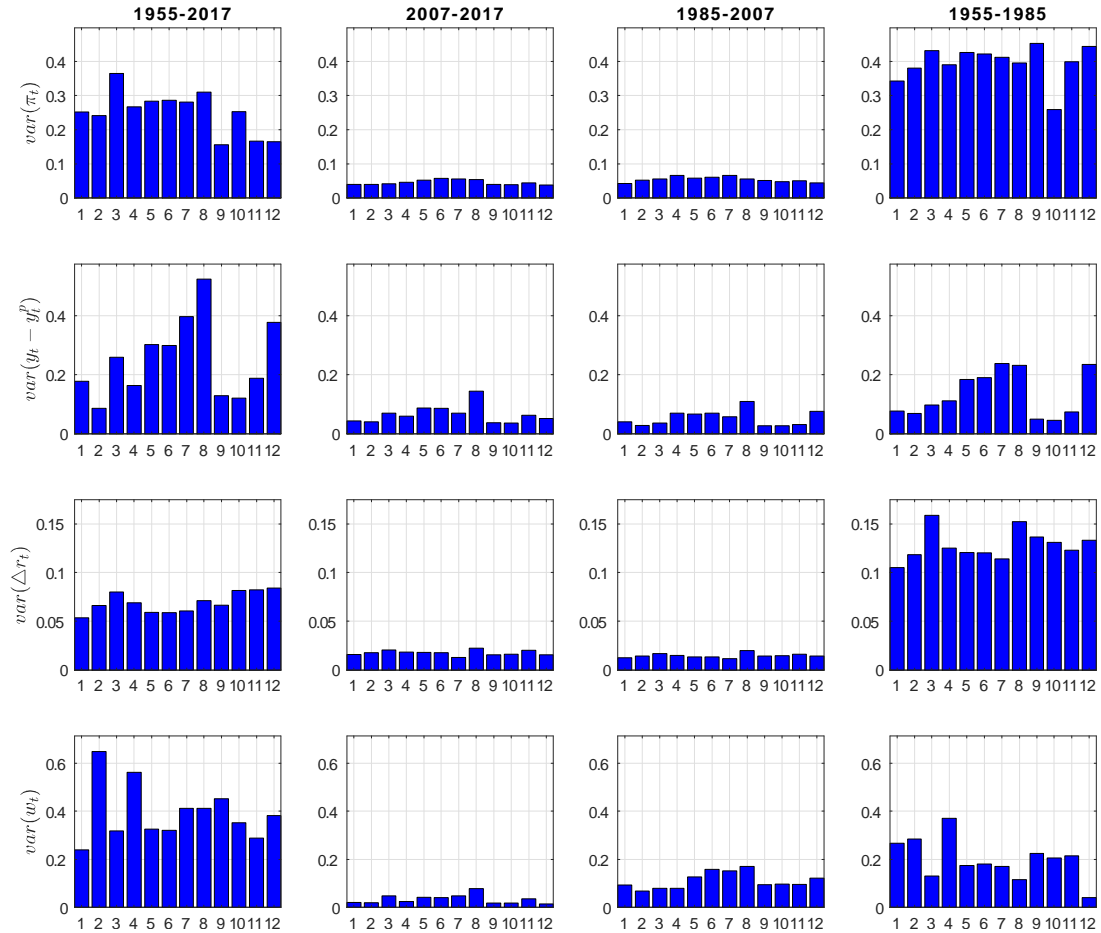


Figure 2: Estimated variances of central bank loss function variables, for each period and each rule.

However, changes in the Fed's monetary policy and the stabilization period that occurred during the 1990s explain the low variance of the GM period relative to the 1955-1985 period. Output variances are a somewhat higher during the GFC/ZLB period than during the GM period, while those of the inflation rate come close. The low interest rates of the GFC/ZLB period lead to lower variances of the shadow interest rate differentials during the GFC/ZLB than during the GM period, although the difference is not



large. The variances of wages were also smaller during the GFC/ZLB period than during the GM period.

Interestingly, the output gap exhibits a low level of volatility during the GFC/ZLB, just a bit higher than during the GM (Fig. 2). The low variances of the output gap over the GM period are due to the fact that, as in most DSGE models, the potential output covaries in general with the current output<sup>16</sup> (Kiley, 2013; Coibion et al., 2019).

During the GFC/ZLB, the correlation between the current output (historical) and the potential output (unobservable, based on our estimations) is rather low compared to the one during the GM. The low output gap variances during the GFC/ZLB are essentially due to the low variances exhibited by both the current and potential outputs over most of the sample after 2009Q2, if not from 2008Q1 to 2009Q1.

Second, we compute ad hoc loss functions based on Eq. 2. The following heatmaps (Tables 4 and 5) present the best (white shading) to the worst (black shading) loss functions in percentage variance for each line.

The loss increases for all rules and for all periods when the weight on the variance of one variable included in the loss functions increases. This is directly related to the linear quadratic functional form of the central bank loss function. The ranking of the monetary policy rules follows from the value of the loss function given by each line, and this ranking changes with respect to the weighting scheme allowed by the central banker's preferences.

When considering the full sample (Table 4, left panel), there is no clear result, with the best rule (rules 9 and 12) being especially sensitive to the values of  $\lambda_w$ .

Rules 9 and 10 (Table 4, right panel) lead to the lowest losses over the GFC/ZLB period, but rule 12 leads to nearly identical results.

Over the GM period (Table 5, left panel) rule 2 often dominates, but rules 1, 9 and 10 come close.

The results vary somewhat when considering the 1955-1985 period (Table 5, right panel), where rule 10 clearly dominates.

Over the 1955-2017 period, generally rule 11 dominates. Yet if the central bank does not pay attention to wage inflation, rule 9 leads to the best results.

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<sup>16</sup>The same type of result comes from some from non-DSGE models, for instance Fernald (2015).

$\lambda_y=0.0 \lambda_r=0.0 \lambda_w=0.0$	25.1	24.1	36.4	26.7	28.3	28.6	28.1	31.0	15.6	25.3	16.6	16.5	3.9	3.9	4.1	4.5	5.1	5.7	5.5	5.3	3.9	3.8	4.4	3.8
$\lambda_y=0.5 \lambda_r=0.0 \lambda_w=0.0$	34.1	28.4	49.4	34.8	43.5	43.6	47.9	57.2	22.1	31.3	26.1	35.4	6.1	5.9	7.6	7.5	9.5	10.0	9.0	12.5	5.8	5.7	7.6	6.4
$\lambda_y=1.0 \lambda_r=0.0 \lambda_w=0.0$	43.0	32.7	62.4	43.0	58.6	58.5	67.8	83.4	28.5	37.3	35.5	54.3	8.2	7.9	11.1	10.5	13.8	14.3	12.5	19.7	7.6	7.5	10.7	9.0
$\lambda_y=0.0 \lambda_r=0.5 \lambda_w=0.0$	27.8	27.4	40.4	30.1	31.3	31.5	31.1	34.5	19.0	29.4	20.8	20.7	4.7	4.8	5.1	5.4	6.0	6.6	6.1	6.4	4.7	4.6	5.4	4.5
$\lambda_y=0.5 \lambda_r=0.5 \lambda_w=0.0$	36.7	31.7	53.4	38.3	46.4	46.5	50.9	60.7	25.4	35.4	30.2	39.6	6.8	6.8	8.6	8.4	10.4	10.9	9.6	13.6	6.5	6.5	8.6	7.1
$\lambda_y=1.0 \lambda_r=0.5 \lambda_w=0.0$	45.6	36.0	66.4	46.4	61.5	61.5	70.8	86.9	31.8	41.4	39.6	58.5	9.0	8.8	12.1	11.4	14.8	15.2	13.1	20.8	8.4	8.3	11.8	9.7
$\lambda_y=0.0 \lambda_r=1.0 \lambda_w=0.0$	30.5	30.7	44.4	33.5	34.2	34.4	34.1	38.1	22.3	33.4	24.9	24.9	5.5	5.7	6.2	6.3	6.9	7.5	6.8	7.5	5.5	5.5	6.4	5.3
$\lambda_y=0.5 \lambda_r=1.0 \lambda_w=0.0$	39.4	35.0	57.4	41.7	49.4	49.4	54.0	64.3	28.7	39.5	34.3	43.8	7.6	7.7	9.7	9.3	11.3	11.8	10.3	14.7	7.3	7.3	9.6	7.9
$\lambda_y=1.0 \lambda_r=1.0 \lambda_w=0.0$	48.3	39.3	70.4	49.9	64.5	64.4	73.8	90.5	35.2	45.5	43.7	62.7	9.8	9.7	13.2	12.3	15.7	16.1	13.8	22.0	9.2	9.1	12.8	10.5
$\lambda_y=0.0 \lambda_r=0.0 \lambda_w=0.5$	37.1	56.6	52.3	54.8	44.6	44.6	48.6	51.6	38.2	42.8	31.1	35.6	4.9	4.8	6.4	5.6	7.2	7.7	7.8	9.1	4.7	4.7	6.1	4.4
$\lambda_y=0.5 \lambda_r=0.0 \lambda_w=0.5$	46.0	60.9	65.3	62.9	59.7	59.6	68.5	77.7	44.6	48.9	40.5	54.5	7.1	6.8	9.9	8.6	11.5	12.0	11.3	16.4	6.6	6.5	9.3	7.0
$\lambda_y=1.0 \lambda_r=0.0 \lambda_w=0.5$	54.9	65.2	78.3	71.1	74.8	74.6	88.4	103.9	51.1	54.9	49.9	73.4	9.2	8.9	13.4	11.6	15.9	16.3	14.8	23.6	8.5	8.4	12.5	9.6
$\lambda_y=0.0 \lambda_r=0.5 \lambda_w=0.5$	39.8	59.9	56.3	58.2	47.5	47.6	51.7	55.1	41.5	46.9	35.2	39.8	5.7	5.7	7.5	6.6	8.1	8.6	8.5	10.3	5.5	5.5	7.1	5.2
$\lambda_y=0.5 \lambda_r=0.5 \lambda_w=0.5$	48.7	64.2	69.3	66.4	62.7	62.5	71.5	81.3	48.0	53.0	44.6	58.7	7.9	7.7	11.0	9.6	12.4	12.9	11.9	17.5	7.4	7.3	10.3	7.8
$\lambda_y=1.0 \lambda_r=0.5 \lambda_w=0.5$	57.6	68.5	82.3	74.5	77.8	77.5	91.4	107.5	54.4	59.0	54.0	77.6	10.0	9.7	14.5	12.5	16.8	17.2	15.4	24.7	9.3	9.2	13.5	10.4
$\lambda_y=0.0 \lambda_r=1.0 \lambda_w=0.5$	42.5	63.2	60.3	61.6	50.5	50.5	54.7	58.7	44.8	51.0	39.3	44.0	6.5	6.6	8.5	7.5	9.0	9.5	9.1	11.4	6.3	6.3	8.1	6.0
$\lambda_y=0.5 \lambda_r=1.0 \lambda_w=0.5$	51.4	67.5	73.3	69.8	65.6	65.5	74.6	84.8	51.3	57.0	48.7	62.9	8.6	8.6	12.0	10.5	13.3	13.8	12.6	18.6	8.2	8.1	11.3	8.6
$\lambda_y=1.0 \lambda_r=1.0 \lambda_w=0.5$	60.3	71.8	86.3	78.0	80.7	80.5	94.4	111.0	57.7	63.1	58.1	81.8	10.8	10.6	15.5	13.5	17.7	18.1	16.1	25.8	10.0	10.0	14.5	11.2
$\lambda_y=0.0 \lambda_r=0.0 \lambda_w=1.0$	49.1	89.0	88.2	82.8	60.8	60.7	69.2	72.1	60.7	60.4	45.5	54.7	5.9	5.7	8.8	6.8	9.2	9.7	10.1	13.0	5.6	5.5	7.9	5.1
$\lambda_y=0.5 \lambda_r=0.0 \lambda_w=1.0$	58.0	93.3	81.2	91.0	76.0	75.7	89.1	98.3	67.2	66.4	54.9	73.6	8.1	7.7	12.3	9.8	13.6	14.0	13.6	20.2	7.5	7.3	11.0	7.7
$\lambda_y=1.0 \lambda_r=0.0 \lambda_w=1.0$	66.9	97.6	94.2	99.2	91.1	90.7	108.9	124.5	73.6	72.5	64.3	92.5	10.2	9.8	15.8	12.8	17.9	18.3	17.1	27.4	9.3	9.2	14.2	10.3
$\lambda_y=0.0 \lambda_r=0.5 \lambda_w=1.0$	51.7	92.3	72.3	86.3	63.8	63.6	72.3	75.7	64.1	64.5	49.6	58.9	6.7	6.6	9.8	7.7	10.1	10.6	10.8	14.1	6.4	6.3	8.9	5.9
$\lambda_y=0.5 \lambda_r=0.5 \lambda_w=1.0$	60.6	96.6	85.2	94.4	78.9	78.6	92.1	101.9	70.5	70.5	59.0	77.8	8.9	8.6	13.3	10.7	14.5	14.9	14.3	21.3	8.2	8.2	12.0	8.5
$\lambda_y=1.0 \lambda_r=0.5 \lambda_w=1.0$	69.5	100.9	98.2	102.6	94.0	93.6	112.0	128.1	77.0	76.5	68.5	96.7	11.0	10.7	16.8	13.7	18.8	19.2	17.8	28.5	10.1	10.0	15.2	11.1
$\lambda_y=0.0 \lambda_r=1.0 \lambda_w=1.0$	54.4	95.6	76.3	89.7	66.7	66.6	75.3	79.2	67.4	68.6	53.7	63.1	7.5	7.5	10.8	8.6	11.0	11.5	11.4	15.2	7.1	7.1	9.9	6.7
$\lambda_y=0.5 \lambda_r=1.0 \lambda_w=1.0$	63.3	99.9	89.3	97.9	81.9	81.5	95.1	105.4	73.8	74.6	63.2	82.0	9.7	9.5	14.3	11.6	15.4	15.8	14.9	22.4	9.0	9.0	13.1	9.3
$\lambda_y=1.0 \lambda_r=1.0 \lambda_w=1.0$	72.2	104.2	102.3	106.0	97.0	96.5	115.0	131.6	80.3	80.6	72.6	100.9	11.8	11.5	17.8	14.6	19.7	20.1	18.4	29.6	10.9	10.8	16.2	11.9
	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12

Table 4: Central bank losses, for each rule (1 to 12), between 1955 and 2007 (left panel) and 2007 and 2017 (right panel). The shading scheme is defined separately in relation to each line. The lighter the shading is, the smaller the loss.

$\lambda_y=0.0 \lambda_r=0.0 \lambda_w=0.0$	4.2	5.2	5.5	6.6	5.8	6.0	6.5	5.5	5.0	4.7	5.0	4.3
$\lambda_y=0.5 \lambda_r=0.0 \lambda_w=0.0$	6.2	6.6	7.3	10.1	9.1	9.5	9.4	11.0	6.4	6.1	6.6	8.2
$\lambda_y=1.0 \lambda_r=0.0 \lambda_w=0.0$	8.2	8.0	9.2	13.6	12.5	13.1	12.3	16.5	7.8	7.4	8.1	12.0
$\lambda_y=0.0 \lambda_r=0.5 \lambda_w=0.0$	4.8	5.9	6.3	7.3	6.5	6.7	7.1	6.5	5.7	5.5	5.8	5.1
$\lambda_y=0.5 \lambda_r=0.5 \lambda_w=0.0$	6.8	7.3	8.2	10.8	9.8	10.2	10.0	12.0	7.1	6.8	7.4	8.9
$\lambda_y=1.0 \lambda_r=0.5 \lambda_w=0.0$	8.8	8.7	10.0	14.3	13.2	13.7	12.9	17.5	8.5	8.2	9.0	12.7
$\lambda_y=0.0 \lambda_r=1.0 \lambda_w=0.0$	5.4	6.6	7.2	8.1	7.1	7.3	7.7	7.5	6.4	6.2	6.6	5.8
$\lambda_y=0.5 \lambda_r=1.0 \lambda_w=0.0$	7.4	8.0	9.0	11.6	10.5	10.9	10.6	13.0	7.8	7.5	8.2	9.6
$\lambda_y=1.0 \lambda_r=1.0 \lambda_w=0.0$	9.5	9.4	10.8	15.1	13.8	14.4	13.5	18.5	9.2	8.9	9.8	13.4
$\lambda_y=0.0 \lambda_r=0.0 \lambda_w=0.5$	8.8	8.5	9.4	10.5	12.1	13.9	14.2	14.1	9.7	9.5	9.7	10.4
$\lambda_y=0.5 \lambda_r=0.0 \lambda_w=0.5$	10.8	9.9	11.2	14.0	15.5	17.4	17.1	19.5	11.1	10.9	11.3	14.2
$\lambda_y=1.0 \lambda_r=0.0 \lambda_w=0.5$	12.8	11.3	13.0	17.5	18.8	21.0	19.9	25.0	12.4	12.2	12.9	18.1
$\lambda_y=0.0 \lambda_r=0.5 \lambda_w=0.5$	9.4	9.3	10.2	11.2	12.8	14.6	14.7	15.0	10.4	10.2	10.6	11.1
$\lambda_y=0.5 \lambda_r=0.5 \lambda_w=0.5$	11.4	10.7	12.1	14.7	16.1	18.1	17.6	20.5	11.8	11.6	12.1	14.9
$\lambda_y=1.0 \lambda_r=0.5 \lambda_w=0.5$	13.5	12.1	13.9	18.2	19.5	21.6	20.5	26.0	13.2	13.0	13.7	18.8
$\lambda_y=0.0 \lambda_r=1.0 \lambda_w=0.5$	10.0	10.0	11.1	12.0	13.4	15.2	15.3	16.0	11.1	11.0	11.4	11.8
$\lambda_y=0.5 \lambda_r=1.0 \lambda_w=0.5$	12.0	11.4	12.9	15.5	16.8	18.8	18.2	21.5	12.5	12.3	12.9	15.7
$\lambda_y=1.0 \lambda_r=1.0 \lambda_w=0.5$	14.1	12.8	14.7	19.0	20.2	22.3	21.1	27.0	13.9	13.7	14.5	19.5
$\lambda_y=0.0 \lambda_r=0.0 \lambda_w=1.0$	13.4	11.9	13.3	14.4	18.4	21.8	21.8	22.6	14.3	14.3	14.5	16.5
$\lambda_y=0.5 \lambda_r=0.0 \lambda_w=1.0$	15.4	13.3	15.1	17.9	21.8	25.3	24.7	28.1	15.7	15.7	16.1	20.3
$\lambda_y=1.0 \lambda_r=0.0 \lambda_w=1.0$	17.4	14.7	16.9	21.4	25.1	28.9	27.6	33.5	17.1	17.0	17.6	24.1
$\lambda_y=0.0 \lambda_r=0.5 \lambda_w=1.0$	14.0	12.6	14.1	15.1	19.1	22.5	22.4	23.6	15.1	15.0	15.3	17.2
$\lambda_y=0.5 \lambda_r=0.5 \lambda_w=1.0$	16.0	14.0	15.9	18.6	22.4	26.0	25.3	29.0	16.4	16.4	16.9	21.0
$\lambda_y=1.0 \lambda_r=0.5 \lambda_w=1.0$	18.1	15.4	17.8	22.1	25.8	29.5	28.1	34.5	17.8	17.8	18.4	24.8
$\lambda_y=0.0 \lambda_r=1.0 \lambda_w=1.0$	14.6	13.3	15.0	15.9	19.8	23.2	22.9	24.6	15.8	15.8	16.1	17.9
$\lambda_y=0.5 \lambda_r=1.0 \lambda_w=1.0$	16.7	14.7	16.8	19.4	23.1	26.7	25.8	30.0	17.2	17.1	17.7	21.7
$\lambda_y=1.0 \lambda_r=1.0 \lambda_w=1.0$	18.7	16.1	18.6	22.9	26.5	30.2	28.7	35.5	18.5	18.5	19.2	25.6
	1	2	3	4	5	6	7	8	9	10	11	12
	34.2	37.9	43.2	38.9	42.6	42.2	41.2	39.6	45.3	25.8	39.9	44.4
	38.0	41.4	48.1	44.5	51.8	51.7	53.2	51.2	47.8	28.1	43.6	56.2
	41.9	44.8	53.0	50.1	61.1	61.2	65.1	62.8	50.2	30.4	47.3	68.0
	39.4	43.9	51.1	45.2	48.7	48.2	46.9	47.2	52.1	32.4	46.1	51.1
	43.3	47.3	56.0	50.8	57.9	57.7	58.9	58.9	54.6	34.7	49.8	62.9
	47.1	50.8	60.9	56.4	67.1	67.2	70.8	70.5	57.1	37.0	53.5	74.7
	44.7	49.8	59.1	51.5	54.7	54.2	52.6	54.9	59.0	38.9	52.2	57.7
	48.6	53.3	64.0	57.1	63.9	63.7	64.6	66.5	61.4	41.3	55.9	69.5
	52.4	56.7	68.9	62.7	73.1	73.2	76.5	78.1	63.9	43.6	59.6	81.3
	47.5	52.2	49.7	57.5	51.4	51.2	49.7	45.3	56.5	36.1	50.7	46.4
	51.3	55.6	54.6	63.1	60.6	60.7	61.7	56.9	59.0	38.4	54.4	58.2
	55.2	59.1	59.5	68.7	69.8	70.2	73.6	68.5	61.5	40.7	58.1	70.0
	52.8	58.1	57.6	63.7	57.4	57.2	55.5	52.9	63.4	42.6	56.8	53.0
	56.6	61.6	62.5	69.3	66.6	66.7	67.4	64.6	65.8	45.0	60.5	64.8
	60.5	65.0	67.4	74.9	75.8	76.2	79.3	76.2	68.3	47.3	64.2	76.6
	58.0	64.1	65.6	70.0	63.4	63.3	61.2	60.6	70.2	49.2	63.0	59.7
	61.9	67.5	70.5	75.6	72.7	72.8	73.1	72.2	72.7	51.5	66.7	71.5
	65.7	70.9	75.4	81.2	81.9	82.3	85.1	83.8	75.1	53.8	70.4	83.3
	60.8	66.4	56.2	76.0	60.1	60.3	58.3	51.0	67.7	46.3	61.4	48.3
	64.7	69.9	61.1	81.6	69.3	69.8	70.2	62.6	70.2	48.7	65.1	60.1
	68.5	73.3	66.0	87.2	78.5	79.3	82.2	74.2	72.7	51.0	68.8	71.9
	66.1	72.4	64.1	82.3	66.1	66.3	64.0	58.6	74.6	52.9	67.6	55.0
	69.9	75.8	69.0	87.9	75.4	75.8	75.9	70.3	77.1	55.2	71.3	66.8
	73.8	79.2	73.9	93.5	84.6	85.3	87.9	81.9	79.5	57.5	75.0	78.6
	71.4	78.3	72.1	88.6	72.2	72.3	69.7	66.3	81.4	59.5	73.7	61.7
	75.2	81.7	77.0	94.2	81.4	81.8	81.7	77.9	83.9	61.8	77.4	73.5
	79.0	85.2	81.9	99.8	90.6	91.3	93.6	89.5	86.4	64.1	81.1	85.3
	1	2	3	4	5	6	7	8	9	10	11	12

Table 5: Central bank losses, for each rule (1 to 12), between 1985 and 2007 (left panel) and 1955 and 1985 (right panel). The shading scheme is defined separately in relation to each line. The lighter the shading is, the smaller the loss.

Over the GFC/ZLB period, such sensitivity to wage inflation is low, and the ranking of rules does not particularly depend on taking wages into consideration. The sensitivity of the results is also low with respect to the values of  $\lambda_y$  and  $\lambda_r$ . The same can be said during the 1955-1985 period and even during the GM period, albeit to a somewhat lesser extent. Interestingly, in all periods, the change in the loss is minor for a given  $\lambda_y$  when  $\lambda_r$  changes, compared to the change in the loss for a given  $\lambda_r$  when  $\lambda_y$  changes. One can interpret this result in light of the interest rate smoothing assumption. Most of the monetary policy rules used in the literature assume interest rate smoothing, as we do. This smoothing implies that the central bank already minimizes the variances in the interest rate differential over time, hence the small gain generated by changing the interest rate differential coefficient in the central bank loss function for a given  $\lambda_y$  or  $\lambda_w$ .

From all these observations, it can be inferred that during the exceptional GFC/ZLB period, the Fed would have minimized its loss by following an NGDP rule in levels, especially rules 9 and 10. During this period, rule 12 performs better under a less credible configuration ( $\lambda_y = 0$ ). However, had it employed Taylor-type rules 1 and 2, the difference in terms of loss would have been minor. Over more stable periods such as the GM period, the central bank would have minimized its losses with a Taylor-type rule, especially rules 1 and 2, but NGDP in level rules (rules 9 and 10) would have led to nearly identical results.

## 5.2 Forecasted loss function

As noted above, a central banker may want to minimize a forecasted loss function based on the dynamics of the model of the economy he uses.

The Bayesian estimation procedure we use allows us to compute the distribution of out-of-sample forecasts while taking into account the uncertainty about parameters and shocks. We use these point forecasts (3 years ahead) to draw the price inflation, output-gap, nominal interest rate differential and wage inflation posterior variances to compute the various forecasted loss functions.

The out-of-sample forecasted losses over a three-year out-of-sample period are presented in Tables 6 and 7. They are based on the estimation of the model with the various monetary rules over the full sample period and over each sub-period.

$\lambda_y=0.0 \lambda_r=0.0 \lambda_w=0.0$	15.1	18.7	21.3	18.9	17.0	18.1	16.4	17.7	13.4	15.7	14.1	13.8
$\lambda_y=0.5 \lambda_r=0.0 \lambda_w=0.0$	19.1	20.9	26.0	21.8	22.2	23.0	22.1	26.7	16.4	18.3	17.9	19.7
$\lambda_y=1.0 \lambda_r=0.0 \lambda_w=0.0$	23.2	23.1	30.8	24.7	27.3	27.9	27.9	35.7	19.3	21.0	21.7	25.6
$\lambda_y=0.0 \lambda_r=0.5 \lambda_w=0.0$	17.4	21.5	24.7	21.8	19.6	20.6	19.0	20.8	16.3	19.0	17.5	17.0
$\lambda_y=0.5 \lambda_r=0.5 \lambda_w=0.0$	21.4	23.7	29.4	24.7	24.7	25.5	24.8	29.8	19.2	21.7	21.3	22.9
$\lambda_y=1.0 \lambda_r=0.5 \lambda_w=0.0$	25.4	25.9	34.1	27.7	29.8	30.4	30.5	38.8	22.1	24.3	25.1	28.8
$\lambda_y=0.0 \lambda_r=1.0 \lambda_w=0.0$	19.6	24.3	28.0	24.7	22.2	23.1	21.6	23.8	19.1	22.3	20.9	20.2
$\lambda_y=0.5 \lambda_r=1.0 \lambda_w=0.0$	23.6	26.5	32.8	27.6	27.3	28.0	27.4	32.9	22.0	25.0	24.7	26.1
$\lambda_y=1.0 \lambda_r=1.0 \lambda_w=0.0$	27.7	28.8	37.5	30.6	32.4	32.9	33.1	41.9	24.9	27.6	28.5	32.0
$\lambda_y=0.0 \lambda_r=0.0 \lambda_w=0.5$	32.3	34.0	39.5	31.1	29.8	32.0	27.5	26.7	25.9	30.9	37.1	17.6
$\lambda_y=0.5 \lambda_r=0.0 \lambda_w=0.5$	36.3	36.3	44.2	34.1	34.9	36.9	33.3	35.7	28.8	33.6	40.9	23.5
$\lambda_y=1.0 \lambda_r=0.0 \lambda_w=0.5$	40.3	38.5	48.9	37.0	40.0	41.7	39.1	44.7	31.7	36.3	44.7	29.4
$\lambda_y=0.0 \lambda_r=0.5 \lambda_w=0.5$	34.5	36.8	42.9	34.0	32.4	34.5	30.2	29.8	28.7	34.3	40.5	20.8
$\lambda_y=0.5 \lambda_r=0.5 \lambda_w=0.5$	38.5	39.1	47.6	37.0	37.5	39.3	35.9	38.8	31.6	36.9	44.3	26.7
$\lambda_y=1.0 \lambda_r=0.5 \lambda_w=0.5$	42.6	41.3	52.3	39.9	42.6	44.2	41.7	47.8	34.5	39.6	48.1	32.6
$\lambda_y=0.0 \lambda_r=1.0 \lambda_w=0.5$	36.8	39.6	46.2	37.0	34.9	37.0	32.8	32.8	31.5	37.6	43.8	24.0
$\lambda_y=0.5 \lambda_r=1.0 \lambda_w=0.5$	40.8	41.9	50.9	39.9	40.0	41.8	38.5	41.8	34.4	40.2	47.7	29.9
$\lambda_y=1.0 \lambda_r=1.0 \lambda_w=0.5$	44.8	44.1	55.7	42.8	45.1	46.7	44.3	50.8	37.3	42.9	51.5	35.8
$\lambda_y=0.0 \lambda_r=0.0 \lambda_w=1.0$	49.4	49.4	57.7	43.4	42.5	45.8	38.7	35.7	38.3	46.2	60.1	21.4
$\lambda_y=0.5 \lambda_r=0.0 \lambda_w=1.0$	53.4	51.6	62.4	46.3	47.6	50.7	44.5	44.7	41.2	48.9	63.9	27.3
$\lambda_y=1.0 \lambda_r=0.0 \lambda_w=1.0$	57.5	53.9	67.1	49.3	52.7	55.6	50.2	53.7	44.1	51.5	67.7	33.2
$\lambda_y=0.0 \lambda_r=0.5 \lambda_w=1.0$	51.7	52.2	61.1	46.3	45.1	48.3	41.3	38.7	41.1	49.5	63.4	24.6
$\lambda_y=0.5 \lambda_r=0.5 \lambda_w=1.0$	55.7	54.4	65.8	49.2	50.2	53.2	47.1	47.7	44.0	52.2	67.3	30.5
$\lambda_y=1.0 \lambda_r=0.5 \lambda_w=1.0$	59.7	56.7	70.5	52.2	55.3	58.1	52.8	56.7	47.0	54.8	71.1	36.4
$\lambda_y=0.0 \lambda_r=1.0 \lambda_w=1.0$	53.9	55.0	64.4	49.2	47.7	50.8	43.9	41.8	43.9	52.8	66.8	27.8
$\lambda_y=0.5 \lambda_r=1.0 \lambda_w=1.0$	57.9	57.3	69.1	52.2	52.8	55.7	49.7	50.8	46.9	55.5	70.6	33.7
$\lambda_y=1.0 \lambda_r=1.0 \lambda_w=1.0$	62.0	59.5	73.8	55.1	57.9	60.6	55.4	59.8	49.8	58.2	74.5	39.6
	1	2	3	4	5	6	7	8	9	10	11	12
	3.7	3.9	3.6	4.1	4.2	4.5	3.5	3.7	3.6	3.6	3.6	3.6
	5.3	5.3	5.5	6.1	6.6	7.0	5.3	10.3	4.9	5.1	5.5	5.6
	6.9	6.8	7.4	8.1	8.9	9.5	7.0	17.0	6.3	6.5	7.4	7.5
	4.3	4.6	4.4	4.9	4.9	5.2	4.1	4.6	4.2	4.3	4.4	4.2
	5.9	6.0	6.3	6.9	7.2	7.7	5.8	11.3	5.6	5.7	6.3	6.2
	7.5	7.5	8.2	8.9	9.6	10.2	7.6	17.9	6.9	7.2	8.2	8.1
	5.0	5.3	5.3	5.7	5.6	5.9	4.6	5.6	4.8	5.0	5.1	4.9
	6.6	6.8	7.2	7.7	7.9	8.4	6.4	12.2	6.2	6.4	7.1	6.8
	8.2	8.2	9.0	9.7	10.3	10.9	8.2	18.8	7.5	7.8	9.0	8.8
	5.0	5.1	5.5	5.5	5.9	6.3	5.0	5.8	4.7	4.8	5.1	4.3
	6.6	6.6	7.3	7.5	8.3	8.7	6.7	12.4	6.1	6.2	7.0	6.2
	8.2	8.0	9.2	9.5	10.7	11.2	8.5	19.1	7.5	7.6	8.9	8.2
	5.7	5.8	6.3	6.3	6.6	7.0	5.5	6.8	5.4	5.4	5.9	4.9
	7.3	7.3	8.2	8.3	9.0	9.5	7.3	13.4	6.7	6.9	7.8	6.9
	8.8	8.7	10.1	10.3	11.4	11.9	9.1	20.0	8.1	8.3	9.7	8.8
	6.3	6.6	7.1	7.1	7.3	7.7	6.1	7.7	6.0	6.1	6.7	5.6
	7.9	8.0	9.0	9.1	9.7	10.2	7.9	14.3	7.4	7.5	8.6	7.5
	9.5	9.4	10.9	11.1	12.0	12.6	9.6	20.9	8.7	9.0	10.5	9.5
	6.4	6.3	7.3	6.8	7.7	8.0	6.4	7.9	5.9	5.9	6.7	5.0
	7.9	7.8	9.2	8.8	10.0	10.5	8.2	14.6	7.3	7.3	8.6	6.9
	9.5	9.2	11.1	10.8	12.4	13.0	10.0	21.2	8.6	8.8	10.5	8.9
	7.0	7.1	8.2	7.6	8.4	8.8	7.0	8.9	6.6	6.6	7.5	5.6
	8.6	8.5	10.1	9.6	10.7	11.2	8.8	15.5	7.9	8.0	9.4	7.6
	10.2	10.0	11.9	11.6	13.1	13.7	10.5	22.1	9.3	9.4	11.3	9.5
	7.6	7.8	9.0	8.4	9.0	9.5	7.6	9.8	7.2	7.2	8.3	6.3
	9.2	9.2	10.9	10.4	11.4	11.9	9.3	16.4	8.5	8.7	10.2	8.2
	10.8	10.7	12.8	12.4	13.8	14.4	11.1	23.1	9.9	10.1	12.1	10.2
	1	2	3	4	5	6	7	8	9	10	11	12

Table 6: Forecasted central bank losses, for each rule (1 to 12), between 2017 and 2019 based on full sample estimates (left panel) and based on 2007-20017 estimates (right panel). The shading scheme is defined separately in relation to each line. The lighter the shading is, the smaller the loss.

$\lambda_y=0.0 \lambda_r=0.0 \lambda_w=0.0$	4.5	4.7	4.6	4.8	4.2	4.4	4.7	4.2	3.8	3.5	3.7	3.6	298	30.3	31.6	28.5	28.6	27.5	26.1	26.4	26.9	21.2	32.6	26.5
$\lambda_y=0.5 \lambda_r=0.0 \lambda_w=0.0$	5.8	5.8	5.6	6.7	6.3	6.4	6.2	6.9	4.9	4.5	4.6	6.0	32.0	32.3	34.8	31.5	32.8	31.5	30.8	32.1	28.7	34.6	32.5	
$\lambda_y=1.0 \lambda_r=0.0 \lambda_w=0.0$	7.1	6.9	6.6	8.6	8.4	8.4	7.7	9.6	5.9	5.6	5.5	8.3	34.1	34.4	38.0	34.6	37.0	35.5	35.5	37.8	30.4	36.7	38.6	
$\lambda_y=0.0 \lambda_r=0.5 \lambda_w=0.0$	5.0	5.3	5.2	5.5	4.8	5.0	5.1	5.1	4.4	4.1	4.3	4.2	34.4	35.6	38.2	33.9	33.8	32.5	30.9	33.0	32.4	37.7	32.1	
$\lambda_y=0.5 \lambda_r=0.5 \lambda_w=0.0$	6.4	6.4	6.2	7.4	6.9	7.0	6.7	7.8	5.4	5.1	5.2	6.5	36.5	37.6	41.4	36.9	38.0	36.6	35.6	38.8	34.2	39.8	38.1	
$\lambda_y=1.0 \lambda_r=0.5 \lambda_w=0.0$	7.7	7.5	7.2	9.2	9.0	9.0	8.2	10.5	6.5	6.1	6.1	8.9	38.7	39.6	44.6	39.9	42.2	40.6	40.3	44.5	35.9	41.8	44.2	
$\lambda_y=0.0 \lambda_r=1.0 \lambda_w=0.0$	5.6	5.9	5.9	6.1	5.4	5.6	5.6	6.0	4.9	4.7	4.9	4.8	39.0	40.8	44.8	39.2	38.9	37.6	35.7	39.7	38.0	42.9	37.7	
$\lambda_y=0.5 \lambda_r=1.0 \lambda_w=0.0$	6.9	7.0	6.9	8.0	7.5	7.6	7.2	8.6	6.0	5.7	5.8	7.1	41.1	42.8	48.0	42.2	43.1	41.6	40.4	45.4	39.7	44.9	43.7	
$\lambda_y=1.0 \lambda_r=1.0 \lambda_w=0.0$	8.2	8.1	7.9	9.9	9.6	9.6	8.7	11.3	7.0	6.7	6.7	9.4	43.2	44.8	51.2	45.2	47.3	45.6	45.1	51.1	41.5	46.9	49.7	
$\lambda_y=0.0 \lambda_r=0.0 \lambda_w=0.5$	7.4	7.3	6.9	7.3	7.7	8.4	8.1	7.9	6.6	6.3	6.5	6.8	40.3	39.4	36.9	39.1	34.7	33.6	31.4	32.1	33.0	39.0	27.8	
$\lambda_y=0.5 \lambda_r=0.0 \lambda_w=0.5$	8.7	8.4	7.9	9.2	9.7	10.4	9.6	10.6	7.6	7.3	7.4	9.1	42.4	41.4	40.1	42.1	38.9	37.6	36.1	37.8	34.8	41.0	33.8	
$\lambda_y=1.0 \lambda_r=0.0 \lambda_w=0.5$	10.0	9.5	8.9	11.0	11.8	12.5	11.1	13.3	8.7	8.3	8.3	11.4	44.5	43.4	43.3	45.1	43.1	41.6	40.9	43.5	36.5	43.0	39.8	
$\lambda_y=0.0 \lambda_r=0.5 \lambda_w=0.5$	7.9	7.9	7.6	7.9	8.2	9.0	8.6	8.8	7.2	6.9	7.1	7.3	44.8	44.6	43.5	44.4	39.9	38.6	36.2	38.7	38.5	44.1	33.4	
$\lambda_y=0.5 \lambda_r=0.5 \lambda_w=0.5$	9.2	9.0	8.6	9.8	10.3	11.0	10.1	11.4	8.2	7.9	8.0	9.7	47.0	46.6	46.7	47.4	44.1	42.6	41.0	44.4	40.3	46.1	39.4	
$\lambda_y=1.0 \lambda_r=0.5 \lambda_w=0.5$	10.6	10.1	9.6	11.7	12.4	13.0	11.6	14.1	9.2	8.9	8.9	12.0	49.1	48.6	49.9	50.5	48.3	46.7	45.7	50.2	42.1	48.1	45.4	
$\lambda_y=0.0 \lambda_r=1.0 \lambda_w=0.5$	8.5	8.4	8.3	8.5	8.8	9.6	9.0	9.6	7.7	7.4	7.7	7.9	49.4	49.9	50.1	49.8	45.0	43.7	41.0	45.3	44.1	49.2	39.0	
$\lambda_y=0.5 \lambda_r=1.0 \lambda_w=0.5$	9.8	9.5	9.3	10.4	10.9	11.6	10.6	12.3	8.8	8.4	8.6	10.2	51.5	51.9	53.3	52.8	49.2	47.7	45.8	51.1	45.8	51.2	45.0	
$\lambda_y=1.0 \lambda_r=1.0 \lambda_w=0.5$	11.1	10.6	10.3	12.3	13.0	13.6	12.1	15.0	9.8	9.4	9.5	12.6	53.6	53.9	56.5	55.8	53.4	51.7	50.5	56.8	47.6	53.3	51.0	
$\lambda_y=0.0 \lambda_r=0.0 \lambda_w=1.0$	10.2	9.9	9.2	9.7	11.1	12.4	11.5	11.6	9.4	9.0	9.3	9.9	50.7	48.4	42.3	49.7	40.8	39.6	36.8	37.8	39.1	45.3	29.1	
$\lambda_y=0.5 \lambda_r=0.0 \lambda_w=1.0$	11.5	11.0	10.3	11.6	13.2	14.5	13.0	14.2	10.4	10.0	10.2	12.2	52.8	50.4	45.5	52.7	45.0	43.7	41.5	43.5	40.9	47.3	35.1	
$\lambda_y=1.0 \lambda_r=0.0 \lambda_w=1.0$	12.9	12.1	11.3	13.5	15.2	16.5	14.5	16.9	11.5	11.0	11.1	14.6	54.9	52.4	48.7	55.7	49.2	47.7	46.2	49.2	42.7	49.4	41.1	
$\lambda_y=0.0 \lambda_r=0.5 \lambda_w=1.0$	10.8	10.4	9.9	10.3	11.6	13.0	12.0	12.4	10.0	9.6	9.9	10.5	55.3	53.6	48.9	55.0	46.0	44.7	41.6	44.4	44.7	50.4	34.7	
$\lambda_y=0.5 \lambda_r=0.5 \lambda_w=1.0$	12.1	11.5	10.9	12.2	13.7	15.1	13.5	15.1	11.0	10.6	10.8	12.8	57.4	55.6	52.1	58.0	50.2	48.7	46.3	50.1	46.4	52.5	40.7	
$\lambda_y=1.0 \lambda_r=0.5 \lambda_w=1.0$	13.4	12.6	11.9	14.1	15.8	17.1	15.0	17.8	12.0	11.6	11.7	15.1	59.5	57.6	55.3	61.0	54.3	52.8	51.0	55.9	48.2	54.5	46.7	
$\lambda_y=0.0 \lambda_r=1.0 \lambda_w=1.0$	11.3	11.0	10.6	10.9	12.2	13.6	12.4	13.3	10.5	10.2	10.5	11.0	59.8	58.9	55.5	60.3	51.1	49.8	46.4	51.0	50.2	55.6	40.3	
$\lambda_y=0.5 \lambda_r=1.0 \lambda_w=1.0$	12.6	12.1	11.6	12.8	14.3	15.6	14.0	16.0	11.6	11.2	11.4	13.4	61.9	60.9	58.7	63.3	55.3	53.8	51.1	56.8	52.0	57.6	46.3	
$\lambda_y=1.0 \lambda_r=1.0 \lambda_w=1.0$	14.0	13.2	12.6	14.7	16.4	17.7	15.5	18.7	12.6	12.2	12.3	15.7	64.1	62.9	61.9	66.4	59.5	57.8	55.8	62.5	53.7	59.6	52.3	
	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12

Table 7: Forecasted central bank losses, for each rule (1 to 12), between 2007 and 2009, based on 1985-2007 estimates (left panel) and between 1985 and 1987, based on 1955-1985 estimates (right panel). The shading scheme is defined separately in relation to each line. The lighter the shading is, the smaller the loss.

The objective here is not to compare to what extent these ex ante forecasted losses diverge from the ex post actual losses over the different sub-periods. A central banker interested in these forecasted values, and who decides today which monetary rule to use, cannot know the ex post values of these losses. He is interested only in minimizing the forecasted values given his model of the economy.

Table 6, left panel, presents the forecasted loss function for 2017-2019 using the full sample. This table shows that rule 9 dominates the other rules when wage inflation is not taken into consideration. Otherwise, rule 12 gives the best results.

Table 6, right panel, presents the forecasted loss function for 2017-2019 using the GFC/ZLB data. Although rule 12 performs well, rules 9 and 10 appear to be optimal if the central banker is interested in realistic forecasted central bank losses ( $\lambda_y > 0$ ).

Table 7, left panel, presents the forecasted loss function for 2007-2009 using the GM period data. Rule 10 (closely followed by 11) is recommended for minimizing central bank losses in the following years (2007-2009).

Table 7, right panel, presents the forecasted loss function for 1985-1987 using the 1955-1985 data. This table clearly shows that rule 10 (again) should be followed to minimize the central bank's loss in the next period. Rule 12 appears to be optimal for less realistic central bank losses ( $\lambda_w = 0$  and  $\lambda_y = 0$ ).

What is remarkable from all these results is that whatever the period used to establish the forecasts, rule 10 (closely followed by 9 and 12) dominates in terms of minimizing the forecasted losses. This is generally the case regardless of the values of the different weights assigned to each variable (inflation, output, wages and interest rate differential).

From this exercise, one can therefore state that the NGDP in level rules clearly dominate the other rules.

If the central bank seeks to minimize such a forecasted loss function in determining its monetary policy, it should choose this type of monetary policy rule.

## 6 Interpretation

### 6.1 Essential facts

Table 8 summarizes our results to capture the essential facts of our exercise.

In terms of fitting the data, the marginal density values show that rule 12 performs better than all others during the GM and GFC/ZLB periods.

	<u>1955-2017</u>	<u>2007-2017</u>	<u>1985-2007</u>	<u>1955-1985</u>
<b>Fitting</b>				
Marginal density	5	12	12	1
<b>Central bank loss</b>				
Current	9,11	9,10 (1,2,12)	2,10 (1,9)	10
Forecasted	9,12	9,12 (10)	10 (11)	10 (12)

Table 8: Summary of the best rule(s) for each criterion. Rules close to the best one(s) are in parentheses.

Rule 1 is best over 1955-1985, while rule 5 dominates from the full sample estimates.

However, for reasons explained in Section 4.2, the values of the marginal densities are not definitive proof that we have the correct ranking of rules. These values constitute an indication as to which rules were more or less followed during the various periods, assuming that the Fed followed a policy rule and that the economy behaved as in the [Smets and Wouters \(2007\)](#) model.

Note that during the GFC/ZLB and the GM periods, the pure NGDP level rule best fits the data but the [Smets and Wouters \(2007\)](#) monetary policy rule is very close during the GM, and even dominates over 1955-1985.

An analysis of the current losses of the central bank generally indicates the superiority of NGDP level rules for all periods, even if the Taylor rule performs as well during the GM.

From Table 8, it can be inferred that during the GFC/ZLB, in-sample fitting and current central bank loss functions indicate that the best performance can be obtained using some NGDP rules. This is not always the case during the other periods.

These results are not intended to prove that the Fed followed any given type of rule in a given period. An explicit rule is only a model that attempts to capture some monetary policy parameters and explain the methodology whereby the central bank determines its interest rate.

The estimates show that an NGDP level rule would be best to minimize the current loss function of the central bank over the various periods (with the Taylor rule performing somewhat better during the GM period).

Table 8 shows that during almost all sub-periods, minimizing the current central bank loss functions does not necessarily lead to one and only one specific preferable monetary policy rule, even if some NGDP in level rules



appear to often be ahead of or at approximately the same ranking as some other rules.

Importantly, the implications that can be drawn from the forecasted losses are illuminating in that respect, showing that the choice of NGDP in level rules would clearly be optimal for minimizing the forecasted losses, whatever the period.

## 6.2 Role of price stickiness and indexation

We examine the coefficients of two important variables of the core model, the degree of price stickiness ( $\xi_p$ ) and the degree of indexation to past prices ( $\iota_p$ ), to better understand why some monetary rules perform better than others over the different periods.<sup>17</sup> Of course, all the coefficients of the models impact the empirical results (approximately 20 parameters, in addition to those in the monetary rules), but it would be particularly cumbersome to deal with all of them.

We choose these two parameters because of their significance in terms of monetary policy effectiveness. As noted by [Schmitt-Grohe and Uribe \(2004\)](#), price-related parameters, such as the degrees of price stickiness and indexation, affect the reaction of monetary policy after a cost-push shock. They also affect the transmission channel of a monetary policy shock to price dynamics. According to [Woodford \(2010\)](#), the variance of such a reaction is determinant in assessing optimal monetary policy.

Let us first examine whether a pattern exists between the different rules during the same period with respect to fitting, central bank losses and forecasted central bank losses with the help of [Table 9](#).

In terms of fitting, over the full sample period, no obvious pattern appears with respect to the degree of price stickiness for rule 5 (the best performer), but the value of the degree of price indexation is the second-highest value in this case. This may mean that during the full period, this last variable played a more significant role than under the other rules in explaining the best fit.

Now, and through the end of this section, we will focus on what is the most interesting exercise: the analysis of the coefficients concerning the GFC/ZLB and the GM periods and the comparison of the results between those periods.

In terms of fit, as far as the GFC/ZLB is concerned, and for the best fitting rule (rule 12), the degree of price stickiness does not exhibit any particular characteristics, but the degree of indexation to past prices is among the

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<sup>17</sup>For further details about these coefficients and the core model, please refer to our online appendix.

Sample	Rule											
	1	2	3	4	5	6	7	8	9	10	11	12
$\xi_p$												
1955-2017	0.803 (0.031)	0.691 (0.034)	0.608 (0.029)	0.646 (0.029)	0.747 (0.028)	0.737 (0.023)	0.737 (0.024)	0.762 (0.023)	0.909 (0.011)	0.873 (0.014)	0.730 (0.013)	0.914 (0.013)
									<b>C,F</b>		<b>C</b>	<b>F</b>
2007-2017	0.879 (0.023)	0.861 (0.025)	0.902 (0.018)	0.841 (0.029)	0.896 (0.021)	0.893 (0.007)	0.908 (0.017)	0.897 (0.012)	0.866 (0.032)	0.863 (0.029)	0.895 (0.021)	0.875 (0.026)
									<b>C,F</b>	<b>C</b>		<b>F</b>
1985-2007	0.875 (0.031)	0.859 (0.029)	0.869 (0.026)	0.734 (0.025)	0.871 (0.020)	0.877 (0.023)	0.872 (0.020)	0.878 (0.020)	0.905 (0.016)	0.907 (0.017)	0.905 (0.015)	0.909 (0.017)
		<b>C</b>								<b>C,F</b>		
1955-1985	0.588 (0.037)	0.566 (0.049)	0.569 (0.043)	0.529 (0.021)	0.568 (0.030)	0.567 (0.033)	0.576 (0.039)	0.575 (0.039)	0.695 (0.033)	0.750 (0.038)	0.632 (0.036)	0.716 (0.051)
										<b>C,F</b>		
$l_p$												
1955-2017	0.247 (0.055)	0.206 (0.042)	0.151 (0.067)	0.216 (0.077)	0.275 (0.052)	0.291 (0.041)	0.197 (0.051)	0.269 (0.057)	0.148 (0.035)	0.211 (0.033)	0.243 (0.052)	0.166 (0.030)
									<b>C,F</b>		<b>C</b>	<b>F</b>
2007-2017	0.288 (0.069)	0.292 (0.081)	0.248 (0.066)	0.278 (0.068)	0.274 (0.050)	0.267 (0.049)	0.234 (0.107)	0.254 (0.053)	0.305 (0.082)	0.305 (0.093)	0.291 (0.119)	0.303 (0.080)
									<b>C,F</b>	<b>C</b>		<b>F</b>
1985-2007	0.275 (0.086)	0.243 (0.043)	0.244 (0.053)	0.210 (0.161)	0.233 (0.061)	0.239 (0.051)	0.246 (0.076)	0.246 (0.049)	0.347 (0.073)	0.335 (0.128)	0.305 (0.063)	0.320 (0.054)
		<b>C</b>								<b>C,F</b>		
1955-1985	0.235 (0.047)	0.221 (0.075)	0.228 (0.058)	0.238 (0.086)	0.344 (0.057)	0.333 (0.120)	0.270 (0.083)	0.224 (0.068)	0.207 (0.065)	0.249 (0.055)	0.206 (0.069)	0.227 (0.072)
										<b>C,F</b>		

Table 9: Degree of price stickiness ( $\xi_p$ ) and indexation ( $l_p$ ) posterior estimates for each period and each rule. The corresponding estimated (posterior) standard deviation is in parentheses. The best in-sample fit for each period is marked in gray (Section 4.2). C and F denote the best current and forecasted loss function, respectively.

highest values (close to rules 9 and 10).

Over the GM period, the degree of price stickiness shows the highest value, whereas the degree of indexation to past prices is among the highest.

The analysis of the coefficients concerning current central bank losses during the GFC/ZLB and GM periods leads to some interesting observations. For the 2007-2017 period the coefficients of the price stickiness variable are almost the lowest for rules 9 and 10 (with the coefficients of rule 2 being close), whereas those of the price indexation are the highest, hence the implication that the second variable played a more important role at the margin than the first one. The coefficients of price stickiness and price indexation are the second highest for rule 10 over the GM period, meaning that, again at the margin, these two variables contribute to explaining the superiority of rule 10 over the other rules.

Regarding the comparison of the GFC/ZLB and GM periods, note that the coefficients of price stickiness and price indexation are lower over 2007-2017 than over 1985-2007, meaning that these two parameters played a less significant role, at the margin, over the GFC/ZLB period than over the GM period.

If we follow the same type of analysis with the forecasted losses, the implications are the same as with the current losses for the GFC/ZLB and GM periods.

Nevertheless, when comparing the two periods, the marginal impact of both variables on forecasted losses appear to be somewhat stronger during the GM period than the GFC/ZLB period.

Ultimately, the marginal roles of price stickiness and price indexation vary across periods. The role of both these parameters appears to be higher over the GM than over the GFC/ZLB periods. The changes in these structural parameters over time support the implication that there is a need to regularly renew model estimations to capture changing policy effects.

## 7 Policy implications

Irrespective of the period in question, central bank's objectives are not achieved by one single monetary policy rule with the same weights given to each variable entering a rule. For each period, there is a preferred monetary policy reaction function. In other words, for each type of period (more or less stable, crisis, recovery), a given type of reaction function performs better than others. However, if we consider the current and forecasted loss functions of the central bank, the results indicate the superiority of NGDP rules in levels, except during the Great Moderation where the Taylor rule works better

with the current loss functions (but some NGDP Level rules are close). The forecasted losses yield non-ambiguous results on the matter: whatever the period and the specific loss function used, it is optimal to use the NGDP level-type rules. However, there is no specific empirical rule, i.e., a rule with fixed parameters, that must always be used whatever the period.

Parameter estimates change with respect to the period considered, for any given monetary policy rule. Policy institutions, which base their forecasts and policy recommendations on such models and rules, should refresh their estimates regularly to avoid inaccurate policy conclusions.

In line with [Wieland et al. \(2012\)](#), our analysis demonstrates that central banks should compare several monetary policy rules to base their policy on a broader scope of results than those obtained by only one model or monetary policy rule.

Most of central banks' DSGE models use ad hoc monetary policy rules, such as Taylor-type rules. It is also standard practice to assume that a central bank seeks to minimize a loss function that includes, at least, inflation and output variances. Would this minimization process necessarily lead to a standard Taylor rule? Our results show that this is not necessarily the case. NGDP in level rules are often superior in terms of minimizing a loss function (current or forecasted). However, both of these types of rules may still sometimes be (or close to) compatible.

*In fine*, what is significant is to use a rule, the NGDP level type being probably the most frequently indicated, especially during crisis and unstable periods, but a Taylor-type rule would also perform well, especially during more stable periods. Furthermore, it is necessary to regularly re-estimate the model, and therefore the monetary policy rule parameters, to better fit to the dynamics of the economy.

## 8 Conclusion

The purpose of this paper is to shed light on the effects of different monetary policy rules on the macroeconomic equilibrium. Specifically, we seek to determine, first, which of the various monetary policy rules is most in line with the historical data for the US economy and, second, what policy rule would work best to assist the central bank in achieving its objectives via several loss function measures, current and forecasted.

To conduct this type of analysis, we compare Taylor-type and nominal income rules through the well-known [Smets and Wouters \(2007\)](#) DSGE model.

We consider twelve monetary policy rules. Four are of the Taylor-type, and eight are of the nominal income targeting type (NGDP), either in growth

or levels. We test the model with these various rules through Bayesian estimations from 1955 to 2017, over three different periods: 1955-1985, 1985-2007, and 2007-2017. These sub-periods are selected to capture the impact of policy rules given different economic environments (more or less stable periods, crisis and recovery).

In terms of fit with historical data, the marginal density values suggest that one NGDP level targeting rule exhibits the best fit during the GFC/ZLB and the GM—and an NGDP growth targeting rule is the best fit over the whole sample. A Taylor-type rule is best during the 1955-1985 period.

The results regarding the current losses of the central bank suggest the superiority of NGDP level targeting rules, over all periods except the Great Moderation, when a Taylor-type rule performs better. However, during the GFC/ZLB period, Taylor-type rules yield results that come close to those of NGDP in level rules, and during the GM period, NGDP in level rules lead also to results that come close to those of Taylor-type rules.

The results are even more clearly in favor of NGDP in level rules, whatever the period, when the minimization of a forecasted loss function is used as an instrumental goal of the central banker.

Several policy implications can be drawn.

First, although a central bank's objectives are best achieved by using monetary rules in the decision-making process, these objectives are not achieved by one single empirical rule with the same weights given to each variable entering the rule. For each type of period (more or less stable, crisis, recovery), a specific reaction function performs better than others.

Second, central banks, which base their forecasts and policy recommendations on such models and rules, should refresh their estimates regularly to avoid inaccurate policy decisions.

Third, policy makers should estimate central bank losses (current or forecasted) through several empirical monetary policy rules and models to better assess their interest rate decisions.

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