Switching volatility and nonlinearities in an open economy - Online appendix*

Jonathan Benchimol[†] and Sergey Ivashchenko[‡]

May 12, 2019

Abstract

This online appendix presents a detailed description of the derivations, the variables and the parameters used in our paper.

Keywords: DSGE, Volatility Shocks, Markov Switching, Open Economy, Financial Crisis, Nonlinearities.

JEL Classification: C61, E32, F21, F41.

*This paper does not necessarily reflect the views of the Bank of Israel or the Ministry of Finance of the Russian Federation . We thank Yossi Yakhin, and participants at the 3rd Centre for Economic Policy Research Network on Macroeconomic Modelling and Model Comparison; the 5th Applied Macroeconomics Workshop by the Henan University and International Network for Economic Research; and the 49th Money, Macro, and Finance Research Group conferences for their valuable comments.

†Bank of Israel, Jerusalem, Israel. Corresponding author. Phone: +972-2-6552641. Fax: +972-2-6669407. Email: jonathan.benchimol@boi.org.il

‡IREP (Russian Academy of Sciences), FRI (Ministry of Finance), and Saint-Petersburg State University, Saint Petersburg, Russia.

1 Derivations

Our generic model consists of a symmetric two-country model in which the domestic (d) and foreign (f) households maximize their country-specific utility subject to their country-specific budget constraint, firms maximize their respective benefits, and central banks follow their respective *ad-hoc* Taylor-type rules and budget constraints.

1.1 Domestic households

The following equations are expressed in terms of stationary variables. The relation between initial and stationary variables is logarithmic after normalization by price or/and level of exogenous technology progress.[1]

The domestic households' optimization problem, corresponding to Eq. 1 to Eq. 4 in the paper, expressed in terms of stationary variables is:

$$E_t\left[\sum_{t=0}^{\infty}\varepsilon^u_{d,t-1}\left(\begin{array}{c}\frac{\left(e^{c_{d,t}}-e^{h_{d,C,t}}\right)^{1-1/\sigma_{d,C}}}{1-1/\sigma_{d,C}}-e^{\phi^L_{d,t}}\frac{\left(e^{l_{d,t}}\right)^{1+1/\sigma_{d,L}}}{1+1/\sigma_{d,L}}+e^{\phi^M_{d,t}}\frac{\left(e^{m_{d,t}}\right)^{1-1/\sigma_{d,M}}}{1-1/\sigma_{d,M}}\\ -\frac{\varphi_{d,s,d}}{2}\left(\frac{b_{d,s,d,t}h_{d,C}}{e^{h_{d,C,t}}}-\mu_{d,s,d}\right)^2-\frac{\varphi_{d,l,d}}{2}\left(\frac{b_{d,l,d,t}h_{d,C}}{e^{h_{d,C,t}}}-\mu_{d,L,d}\right)^2\\ -\frac{\varphi_{d,s,f}}{2}\left(\frac{b_{d,s,f,t}h_{d,C}}{e^{h_{d,C,t}}}-\mu_{d,s,f}\right)^2-\frac{\varphi_{d,l,f}}{2}\left(\frac{b_{d,l,f,t}h_{d,C}}{e^{h_{d,C,t}}}-\mu_{d,l,f}\right)^2\end{array}\right)\right]\rightarrow\max_{B,C,L,M} \tag{1}$$

$$\begin{aligned}&e^{c_{d,t}}+e^{m_{d,t}}+b_{d,s,d,t}e^{-r_{d,s,t}}+b_{d,l,d,t}e^{-r_{d,l,t}}+b_{d,s,f,t}e^{f_t-r_{f,s,t}}+b_{d,l,f,t}e^{f_t-r_{f,l,t}}=\\&+b_{d,s,d,t-1}e^{-p_{d,t}-\phi^y_t}+b_{d,l,d,t-1}e^{-p_{d,t}-\phi^y_t}\left(\left(1-s_d\right)e^{-r_{d,l,t}}+s_d\right)+e^{w_{d,t}+l_{d,t}}+\\&b_{d,s,f,t-1}e^{f_t-p_{f,t}-\phi^y_t}+b_{d,l,f,t-1}e^{f_t-p_{f,t}-\phi^y_t}\left(\left(1-s_f\right)e^{-r_{f,l,t}}+s_f\right)+e^{m_{d,t}-p_{d,t}-\phi^y_t}+d_{d,t}\end{aligned} \tag{2}$$

The utility function consists of consumption, with part of the consumption behavior attributed to habits in preferences, labor, and money in the utility. A friction *à la* Rotemberg (1982a) is added in the utility function for each type of bond position.[2] This is required when two agents with different intertemporal preferences are trading the same security, especially bonds.[3]

Note that we use the following external consumption habits in the quadratic costs because the bond position should be expressed in terms of household's consumption.[4]

$$e^{h_{d,C,t}}=h_{d,C}e^{c_{d,t-1}} \tag{3}$$

Eq. 3 describes how consumption habits are formed and parameter $h_{d,C}$ contributes to the rigidity.

The market consists of domestic and foreign one-period bonds and long-term bonds. Long-term bonds pay a share (s_d, s_f) of its nominal each period. Thus, long-term bonds of nominal 1 euro produce s_d euro in the first period, $s_d\left(1-s_d\right)$ in the second, $s_d\left(1-s_d\right)^2$ in the third, and so on.

[1] For instance, $c_{D,t}=\ln\left(C_{D,t}/Z_t\right)$ or $m_{D,t}=\ln\left(M_{D,t}/(Z_tP_{D,t})\right)$. The relation between the exchange rate and the stationary exchange rate (real exchange rate) is $f_t=\ln\left(e_tP_{F,t}/P_{D,t}\right)$. Because the bond position could be negative, we do not take logs. See Section ?? for more details about these transformations.

[2] This is equivalent to a credit-borrowing constraint.

[3] Otherwise, agents could find optimal to have minus infimum position that is not realistic. There is an alternative way to modulate the rigidity by restricting negative values. However, while simple quadratic costs produce smoothed restriction, this would be more complicated.

[4] The use of the consumption habits allow us simplify the optimization with respect to consumption.

The FOCs of the domestic households are

$$- e^{\phi_{d,t}^M} \left(e^{m_{d,t}}\right)^{-1/\sigma_{d,M}} e^{m_{d,t}} - \lambda_{d,t} e^{m_{d,t}} + \lambda_{d,t+1} e^{\phi_{d,t}^u + m_{d,t} - p_{d,t+1} - \phi_{t+1}^y} = 0 \tag{4}$$

$$- e^{\phi_{d,t}^L} \left(e^{l_{d,t}}\right)^{1/\sigma_{d,L}} e^{l_{d,t}} + \lambda_{d,t} e^{l_{d,t} + w_{d,t}} = 0 \tag{5}$$

$$\left(e^{c_{d,t}} - h_{d,C} e^{c_{d,t-1}}\right)^{-1/\sigma_{d,C}} e^{c_{d,t}} - \lambda_{d,t} e^{c_{d,t}} = 0 \tag{6}$$

$$- \varphi_{d,s,d} \left(b_{d,s,d,t} e^{-c_{d,t-1}} - \mu_{d,s,d}\right) e^{-c_{d,t-1}} - \lambda_{d,t} e^{-r_{d,s,t}} + \lambda_{d,t+1} e^{\phi_{d,t}^u - p_{d,t+1} - \phi_{t+1}^y} = 0 \tag{7}$$

$$\begin{aligned} &-\varphi_{d,l,d} \left(b_{d,l,d,t} e^{-c_{d,t-1}} - \mu_{d,l,d}\right) e^{-c_{d,t-1}} - \lambda_{d,t} e^{-r_{d,L,t}} + \\ &+\lambda_{d,t+1} e^{\phi_{d,t}^u - p_{d,t+1} - \phi_{t+1}^y} \left(\left(1 - s_d\right) e^{-r_{d,L,t+1}} + s_d\right) = 0 \end{aligned} \tag{8}$$

$$- \varphi_{d,s,f} \left(b_{d,s,f,t} e^{-c_{d,t-1}} - \mu_{d,s,f}\right) e^{-c_{d,t-1}} - \lambda_{d,t} e^{f_t - r_{f,t}} + \lambda_{d,t+1} e^{\phi_{d,t}^u + f_{t+1} - p_{f,t+1} - \phi_{t+1}^y} = 0 \tag{9}$$

$$\begin{aligned} &-\varphi_{d,l,f} \left(b_{d,l,f,t} e^{-c_{d,t-1}} - \mu_{d,l,f}\right) e^{-c_{d,t-1}} - \lambda_{d,t} e^{f_t - r_{f,l,t}} + \\ &+\lambda_{d,t+1} e^{\phi_{d,t}^u + f_{t+1} - p_{f,t+1} - \phi_{t+1}^y} \left(\left(1 - s_f\right) e^{-r_{f,l,t+1}} + s_f\right) = 0 \end{aligned} \tag{10}$$

These FOCs (Eq. 4 to Eq. 10), which are easier to compute in stationary variables, can be rewritten in nonstationary variables (Eq. 11 to Eq. 17), which are more readable, such as

$$- \varepsilon_{d,t}^M \left(\frac{M_{d,t}}{Z_t P_{d,t}}\right)^{-1/\sigma_{d,M}} \frac{M_{d,t}}{Z_t P_{d,t}} - \lambda_{d,t} \frac{M_{d,t}}{Z_t P_{d,t}} + \frac{\lambda_{d,t+1} Z_t P_{d,t}}{Z_{t+1} P_{d,t+1}} \frac{\varepsilon_{d,t}^u}{\varepsilon_{d,t-1}^u} \frac{M_{d,t}}{Z_t P_{d,t}} = 0 \tag{11}$$

$$- \varepsilon_{d,t}^L \left(L_{d,t}\right)^{1/\sigma_{d,L}} L_{d,t} + \lambda_{d,t} L_{d,t} W_{d,t} = 0 \tag{12}$$

$$\left(\frac{C_{d,t}}{Z_t} - h_{d,C} \frac{C_{d,t-1}}{Z_{t-1}}\right)^{-1/\sigma_{d,C}} \frac{C_{d,t}}{Z_t} - \lambda_{d,t} \frac{C_{d,t}}{Z_t} = 0 \tag{13}$$

$$- \varphi_{d,s,d} \left(\frac{B_{d,s,d,t} Z_{t-1}}{P_{d,t} C_{d,t-1} Z_t} - \mu_{d,s,d}\right) \frac{Z_t}{C_{d,t}} - \lambda_{d,t} \frac{1}{R_{d,s,t}} + \frac{\lambda_{d,t+1} Z_t P_{d,t}}{Z_{t+1} P_{d,t+1}} \frac{\varepsilon_{d,t}^u}{\varepsilon_{d,t-1}^u} = 0 \tag{14}$$

$$\begin{aligned} &-\varphi_{d,l,d} \left(\tfrac{B_{d,l,d,t} Z_{t-1}}{P_{d,t} C_{d,t-1} Z_t} - \mu_{d,l,d}\right) \tfrac{Z_{t-1}}{C_{d,t-1}} - \lambda_{d,t} \tfrac{1}{R_{d,l,t}} + \\ &+ \tfrac{\lambda_{d,t+1} Z_t P_{d,t}}{Z_{t+1} P_{d,t+1}} \tfrac{\varepsilon_{d,t}^u}{\varepsilon_{d,t-1}^u} \left(\left(1 - s_d\right) \tfrac{1}{R_{d,l,t+1}} + s_d\right) = 0 \end{aligned} \tag{15}$$

$$-\varphi_{d,s,f} \left(\frac{B_{d,s,f,t}}{P_{f,t} Z_t} \frac{Z_{t-1}}{C_{d,t-1}} - \mu_{d,s,f}\right) \frac{Z_{t-1}}{C_{d,t-1}} - \lambda_{d,t} \frac{e_{d,t} P_{f,t}}{P_{d,t} R_{f,s,t}} + \frac{\lambda_{d,t+1} Z_t P_{f,t}}{Z_{t+1} P_{f,t+1}} \frac{e_{d,t+1} P_{f,t+1}}{P_{d,t+1}} \frac{\varepsilon_{d,t}^u}{\varepsilon_{d,t-1}^u} = 0 \tag{16}$$

$$\begin{aligned} &-\varphi_{d,l,f} \left(\tfrac{B_{d,l,f,t}}{P_{f,t} Z_t} \tfrac{Z_{t-1}}{C_{d,t-1}} - \mu_{d,l,f}\right) \tfrac{Z_{t-1}}{C_{d,t-1}} - \lambda_{d,t} \tfrac{e_{d,t} P_{f,t}}{P_{d,t} R_{f,l,t}} + \\ &+ \tfrac{\lambda_{d,t+1} Z_t P_{f,t}}{Z_{t+1} P_{f,t+1}} \tfrac{e_{d,t+1} P_{f,t+1}}{P_{d,t+1}} \tfrac{\varepsilon_{d,t}^u}{\varepsilon_{d,t-1}^u} \lambda_{d,t+1} \left(\left(1 - s_f\right) \tfrac{1}{R_{f,l,t+1}} + s_f\right) = 0 \end{aligned} \tag{17}$$

1.2 Foreign households

The foreign household's problem, corresponding to Eq. 1 to Eq. 4 in the paper, is symmetric:

$$E_t \left[\sum_{t=0}^{\infty} \varepsilon_{f,s-1}^u \left(\begin{array}{l} \frac{\left(e^{c_{f,t}} - e^{h_{f,C,t}}\right)^{1-1/\sigma_{f,C}}}{1-1/\sigma_{f,C}} - e^{\phi_{f,t}^L} \frac{\left(e^{l_{f,t}}\right)^{1+1/\sigma_{f,L}}}{1+1/\sigma_{f,L}} + e^{\phi_{f,t}^M} \frac{\left(e^{m_{f,t}}\right)^{1-1/\sigma_{f,M}}}{1-1/\sigma_{f,M}} \\ - \frac{\varphi_{f,s,f}}{2} \left(\frac{b_{f,s,f,t} h_{f,C}}{e^{h_{f,C,t}}} - \mu_{f,s,f}\right)^2 - \frac{\varphi_{f,l,f}}{2} \left(\frac{b_{f,l,f,t} h_{f,C}}{e^{h_{f,C,t}}} - \mu_{f,l,f}\right)^2 \\ - \frac{\varphi_{f,s,d}}{2} \left(\frac{b_{f,s,d,t} h_{f,C}}{e^{h_{f,C,t}}} - \mu_{f,s,d}\right)^2 - \frac{\varphi_{f,l,d}}{2} \left(\frac{b_{f,l,d,t} h_{f,C}}{e^{h_{f,C,t}}} - \mu_{f,l,d}\right)^2 \end{array} \right) \right] \to \max_{B,C,L,M} \tag{18}$$

$$e^{c_{f,t}} + e^{m_{f,t}} + b_{f,s,f,t}e^{-r_{f,s,t}} + b_{f,l,f,t}e^{-r_{f,l,t}} + b_{f,s,d,t}e^{-f_t - r_{d,s,t}} + b_{f,l,d,t}e^{-f_t - r_{d,l,t}} = \\ +b_{f,s,f,t-1}e^{-p_{f,t}-\phi_t^y} + b_{f,l,f,t-1}e^{-p_{f,t}-\phi_t^y}\left(\left(1-s_f\right)e^{-r_{f,l,t}} + s_f\right) + e^{m_{f,t}-p_{f,t}-\phi_t^y} + \\ b_{f,s,d,t-1}e^{-f_t-p_{d,t}-\phi_t^y} + b_{f,l,d,t-1}e^{-f_t-p_{d,t}-\phi_t^y}\left(\left(1-s_d\right)e^{-r_{d,l,t}} + s_d\right) + e^{w_{f,t}+l_{f,t}} + d_{f,t} \tag{19}$$

The main difference between these two optimization problems, Eq. 1 to Eq. 2 and 18 to Eq. 19, resides in the use of the exchange rate variable.

1.3 Domestic firms

The firm's problem with a linear production function and price adjustment costs as in Rotemberg (1982a,b) corresponds to Eq. 5 to Eq. 8 in the paper and can be rewritten as following.

The firm maximizes the following expression

$$\frac{1}{P_{d,t}Z_t}E_t\left[\sum_{s=0}^{\infty}\left(\prod_{k=0}^{s-1}R_{d,s,t+k}\right)^{-1}\left(D_{d,t+s} - \varphi_{d,P}\left(\frac{P_{F,d,t}\left(j\right)}{P_{F,d,t-1}\left(j\right)e^{\overline{p_d}v+(1-v)p_{d,t-1}}} - 1\right)^2 P_{d,t+s}Y_{d,t+s}\right)\right] \tag{20}$$

with respect to D, P, Y, and L, and the following constraints

$$D_{d,t} + W_tL_t\left(j\right) = P_{F,d,t}\left(j\right)Y_{F,d,t}\left(j\right) \tag{21}$$

$$Y_{F,d,t}\left(j\right) = \left(\frac{P_{F,d,t}\left(j\right)}{P_{d,t}}\right)^{-\varepsilon_{d,t}^{\theta}}\omega_dY_{d,t} + \left(\frac{P_{F,d,t}\left(j\right)}{F_tP_{f,t}}\right)^{-\varepsilon_{f,t}^{\theta}}(1-\omega_f)Y_{f,t} \tag{22}$$

$$Y_{F,d,t}\left(j\right) = A_dZ_tL_{d,t}\left(j\right) \tag{23}$$

This problem (Eq. 20 to Eq. 23) can be rewritten in terms of stationary variables (Eq. 24 to Eq. 27).

$$E_t\left[\begin{array}{c}\sum_{s=0}^{\infty} e^{-\sum_{j=1}^{s}\left(r_{d,s,t+j-1}-\phi_{t+j}^y-p_{d,t+j}\right)} \\ \left(d_{d,t+s} - \varphi_{d,P}\left(e^{p_{F,d,t}(j)-p_{F,d,t-1}(j)+p_{d,t}-(\overline{p_d}v+(1-v)p_{d,t-1})} - 1\right)^2 e^{y_{d,t+s}}\right)\end{array}\right] \tag{24}$$

$$d_{d,t} + e^{w_t+l_t(j)} = e^{p_{F,d,t}(j)+y_{F,d,t}(j)} \tag{25}$$

$$e^{y_{F,d,t}(j)} = e^{-\phi_{d,t}^{\theta}p_{F,d,t}(j)+y_{d,t}}\omega_d + (1-\omega_F)e^{-\phi_{f,t}^{\theta}(p_{F,d,t}(j)-f_t)+y_{f,t}} \tag{26}$$

$$e^{y_{F,d,t}(j)} = e^{a_d+l_{d,t}(j)} \tag{27}$$

Production function (Eq. 23) includes a unit root technology process Z_t and a country specific TFP parameter. The firms sell its production on domestic and foreign markets with the same price. The demand function (Eq. 26) results from a usual CES-aggregation.

1.4 Domestic firms

After simplification, the firms FOC is

$$
\begin{aligned}
&e^{p_{F,d,t}(j)}\left(e^{-\phi^{\theta}_{d,t}p_{F,d,t}(j)+y_{d,t}}\omega_d+(1-\omega_f)e^{-\phi^{\theta}_{f,t}(p_{F,d,t}(j)-f_t)+y_{f,t}}\right)+\\
&+\left(e^{p_{F,d,t}(j)}-e^{w_{d,t}-a_d}\right)\left(-\phi^{\theta}_{d,t}e^{-\phi^{\theta}_{d,t}p_{F,d,t}(j)+y_{d,t}}\omega_d-\phi^{\theta}_{f,t}(1-\omega_f)e^{-\phi^{\theta}_{f,t}(p_{F,d,t}(j)-f_t)+y_{f,t}}\right)-\\
&-\varphi_{d,P}\left(e^{p_{F,d,t}(j)-p_{F,d,t-1}(j)+p_{d,t}-(\overline{p_d}v+(1-v)p_{d,t-1})}-1\right)e^{y_{d,t}+p_{F,d,t}(j)}+\\
&+\varphi_{d,P}\left(e^{p_{F,d,t+1}(j)-p_{F,d,t}(j)+p_{d,t+1}-(\overline{p_d}v+(1-v)p_{d,t})}-1\right)e^{y_{d,t+1}+p_{F,d,t}(j)-\left(r_{d,s,t}-\phi^{y}_{t+1}-p_{d,t+1}\right)}=0
\end{aligned}
\tag{28}
$$

which can be rewritten in terms of nonstationary variables

$$
\begin{aligned}
&\frac{P_{F,d,t}(j)}{P_{d,t}}\left(\left(\frac{P_{F,d,t}(j)}{P_{d,t}}\right)^{-\varepsilon^{\theta}_{d,t}}Y_{d,t}\omega_d+(1-\omega_f)\left(\frac{P_{F,d,t}(j)P_{d,t}}{e_{d,t}P_{d,t}P_{f,t}}\right)^{-\varepsilon^{\theta}_{f,t}(p_{F,d,t}(j)-f_t)}Y_{f,t}\right)+\\
&+\left(\frac{P_{F,d,t}(j)}{P_{d,t}}-\frac{w_{d,t}}{A_d}\right)\left(-\varepsilon^{\theta}_{d,t}\left(\frac{P_{F,d,t}(j)}{P_{d,t}}\right)^{-\varepsilon^{\theta}_{d,t}}Y_{d,t}\omega_d-\varepsilon^{\theta}_{f,t}(1-\omega_f)\left(\frac{P_{F,d,t}(j)P_{d,t}}{e_{d,t}P_{d,t}P_{f,t}}\right)^{-\varepsilon^{\theta}_{f,t}\left(p_{F,d,t}(j)-f_t\right)}Y_{f,t}\right)-\\
&-\varphi_{d,P}\left(\frac{P_{F,d,t}(j)}{P_{F,d,t-1}(j)}e^{\cdot-\overline{p_d}v}\left(\frac{P_{d,t-1}}{P_{d,t-2}}\right)^{\cdot-(1-v)}-1\right)Y_{d,t}\frac{P_{F,d,t}(j)}{P_{d,t}}+\\
&+\varphi_{d,P}\left(\frac{P_{F,d,t+1}(j)}{P_{F,d,t}(j)}e^{\cdot-\overline{p_d}v}\left(\frac{P_{d,t}}{P_{d,t-1}}\right)^{\cdot-(1-v)}-1\right)\frac{Y_{d,t+1}Z_{t+1}P_{d,t+1}}{R_{d,s,t}Z_tP_{d,t}}\frac{P_{F,d,t}(j)}{P_{d,t}}=0
\end{aligned}
\tag{29}
$$

1.5 Foreign firms

The foreign firm's problem consists in maximizing

$$
\frac{1}{P_{f,t}Z_t}E_t\left[\sum_{s=0}^{\infty}\left(\prod_{k=0}^{s-1}R_{f,s,t+k}\right)^{-1}\left(D_{f,t+s}-\varphi_{f,P}\left(\frac{P_{F,f,t}(j)}{P_{F,f,t-1}(j)\,e^{\overline{p_f}v+(1-v)p_{f,t-1}}}-1\right)^2P_{f,t+s}Y_{f,t+s}\right)\right]
\tag{30}
$$

with respect to D, P, Y, and L, and the following constraints

$$D_{f,t+s}+W_{f,t+s}L_{f,t+s}(j)=P_{F,f,t}(j)\,Y_{F,f,t+s}(j) \tag{31}$$

$$Y_{F,f,t}(j)=\left(\frac{P_{F,f,t}(j)}{P_{f,t}}\right)^{-z_{f,\theta,t}}\omega_fY_{f,t}+\left(\frac{F_tP_{F,f,t}(j)}{P_t}\right)^{-z_{d,\theta,t}}(1-\omega_d)Y_{d,t} \tag{32}$$

$$Y_{F,f,t}(j)=Z_tA_fL_{f,t}(j) \tag{33}$$

The foreign firms problem (Eq. 30 to Eq. 33) can be rewritten in terms of variables, similar to domestic firms, such as

$$
E_t\left[\sum_{s=0}^{\infty}e^{-\sum_{j=1}^{s}\left(r_{f,s,t+j-1}-\phi^{y}_{t+j}-p_{f,t+j}\right)}\left(d_{f,t+s}-\varphi_{f,P}\left(e^{p_{F,f,t}(j)-P_{F,f,t-1}(j)+p_{f,t}-(\overline{p_f}v+(1-v)p_{f,t-1})}-1\right)^2e^{y_{f,t+s}}\right)\right]
\tag{34}
$$

$$d_{f,t+s}+e^{w_{f,t+s}+l_{f,t+s}(j)}=e^{p_{F,f,t}(j)+y_{F,f,t+s}(j)} \tag{35}$$

$$e^{y_{F,f,t}(j)}=e^{-\phi^{\theta}_{f,t}p_{F,f,t}(j)+y_{f,t}}\omega_f+(1-\omega_d)e^{-\phi^{\theta}_{d,t}(p_{F,f,t}(j)+f_t)+y_{d,t}} \tag{36}$$

$$e^{y_{F,f,t}(j)}=e^{a_f+l_{f,t}(j)} \tag{37}$$

1.6 Governments

The central banks follow a Taylor-type rule and the budget constraints (Eq. 10 to Eq. 11 in the paper) can be rewritten in stationary variables such as

$$r_{d,s,t} = \gamma_{d,R} \left(r_{d,s,t-1}\right) + \left(1 - \gamma_{d,R}\right) \left(\gamma_{d,P} \left(p_{d,t} - \overline{p}\right) + \gamma_{d,Y} \left(y_{d,t} - \overline{y_d}\right) + \gamma_{d,f} \left(f_t - \overline{f}\right)\right) + \phi^R_{d,t} \tag{38}$$

$$b_{d,G,t} e^{-r_{d,s,t}} = b_{d,G,t-1} e^{-p_{d,t} - \phi^y_t} + \left(e^{m_{d,t}} - e^{m_{d,t-1} - p_{d,t} - \phi^y_t}\right) \tag{39}$$

$$r_{f,s,t} = \gamma_{f,R} \left(r_{f,s,t-1}\right) + \left(1 - \gamma_{f,R}\right) \left(\gamma_{f,P} \left(p_{f,t} - \overline{p}\right) + \gamma_{f,Y} \left(y_{f,t} - \overline{y_f}\right) + \gamma_{f,f} \left(f_t - \overline{f}\right)\right) + \phi^R_{f,t} \tag{40}$$

$$b_{f,G,t} e^{-r_{f,s,t}} = b_{f,G,t-1} e^{-p_{f,t} - \phi^y_t} + e^{m_{f,t}} - e^{m_{f,t-1} - p_{f,t} - \phi^y_t} \tag{41}$$

1.7 Equilibrium

The domestic demand, corresponding to Eq. 12 in the paper, consists of consumption only such as

$$y_{d,t} = c_{d,t} \tag{42}$$

$$y_{f,t} = c_{f,t} \tag{43}$$

The aggregate price level, corresponding to Eq. 9 in the paper, consists of a usual CES-aggregation such as

$$1 = e^{p_{F,d,t}\left(1 - \phi^\theta_{d,t}\right)} \left(\omega_d\right) + e^{\left(p_{F,f,t} + f_t\right)\left(1 - \phi^\theta_{d,t}\right)} \left(1 - \omega_d\right) \tag{44}$$

$$1 = e^{p_{F,f,t}\left(1 - \phi^\theta_{f,t}\right)} \left(\omega_f\right) + e^{\left(p_{F,d,t} - f_t\right)\left(1 - \phi^\theta_{f,t}\right)} \left(1 - \omega_f\right) \tag{45}$$

Each bond should be bought by someone (Eq. 13 and Eq. 14 in the paper) which can be rewritten in stationary variables such as

$$b_{d,s,d,t} + b_{f,s,d,t} + b_{d,G,t} = 0 \tag{46}$$

$$b_{d,s,f,t} + b_{f,s,f,t} + b_{f,G,t} = 0 \tag{47}$$

$$b_{d,l,d,t} + b_{f,l,d,t} = 0 \tag{48}$$

$$b_{d,l,f,t} + b_{f,l,f,t} = 0 \tag{49}$$

2 Summary of variables

Variable	Description	Stationary variable
$B_{d,s,d,t}$	Domestic bonds bought by domestic households	$b_{d,s,d,t} = B_{d,s,d,t}/P_{d,t}Z_t$
$B_{d,s,f,t}$	Foreign bonds bought by domestic households	$b_{d,s,f,t} = B_{d,s,f,t}/P_{f,t}Z_t$
$B_{d,G,t}$	Domestic bonds bought by domestic central bank	$b_{d,G,t} = B_{d,G,t}/P_{d,t}Z_t$
$B_{f,s,d,t}$	Domestic bonds bought by foreign households	$b_{f,s,d,t} = B_{f,s,d,t}/P_{d,t}Z_t$
$B_{f,s,f,t}$	Foreign bonds bought by foreign households	$b_{f,s,f,t} = B_{f,s,f,t}/P_{f,t}Z_t$
$B_{f,G,t}$	Domestic bonds bought by foreign central bank	$b_{f,G,t} = B_{f,G,t}/P_{f,t}Z_t$
$B_{d,l,d,t}$	Domestic long-term bonds bought by domestic households	$b_{d,l,d,t} = B_{d,l,d,t}/P_{d,t}Z_t$
$B_{f,l,d,t}$	Domestic long-term bonds bought by foreign households	$b_{f,l,d,t} = B_{f,l,d,t}/P_{d,t}Z_t$
$B_{d,l,f,t}$	Foreign long-term bonds bought by domestic households	$b_{d,l,f,t} = B_{d,l,f,t}/P_{f,t}Z_t$
$B_{f,l,f,t}$	Foreign long-term bonds bought by foreign households	$b_{f,l,f,t} = B_{f,l,f,t}/P_{f,t}Z_t$
$C_{d,t}$	Consumption of domestic households	$c_{d,t} = \ln (C_{d,t}/Z_t)$
$C_{f,t}$	Consumption of foreign households	$c_{f,t} = \ln (C_{f,t}/Z_t)$
$D_{d,t}$	Dividends of domestic firms	$d_{d,t} = D_{d,t}/P_{d,t}Z_t$
$D_{f,t}$	Dividends of foreign firms	$d_{f,t} = D_{f,t}/P_{f,t}Z_t$
$e_{D,t}$	Exchange rate (domestic per 1 unit of foreign currency)	$f_t = \ln (e_{d,t}P_{f,t}/P_{D,t})$
$L_{d,t}$	Domestic labor	$l_{d,t} = \ln (L_{d,t})$
$L_{f,t}$	Foreign labor	$l_{f,t} = \ln (L_{f,t})$
$M_{d,t}$	Money stock holds by domestic household	$m_{d,t} = \ln (M_{d,t}/P_{d,t}Z_t)$
$M_{d,t}$	Money stock holds by domestic household	$m_{d,t} = \ln (M_{d,t}/P_{d,t}Z_t)$
$P_{d,t}$	Aggregate price level on domestic market	$p_{d,t} = \ln (P_{d,t}/P_{d,t-1})$
$P_{f,t}$	Aggregate price level on foreign market	$p_{f,t} = \ln (P_{f,t}/P_{f,t-1})$
$P_{F,d,t}(j)$	Price for goods of domestic firm j	$p_{F,d,t}(j) = \ln (P_{F,d,t}(j)/P_{d,t})$
$P_{F,f,t}(j)$	Price for goods of foreign firm j	$p_{F,f,t}(j) = \ln (P_{F,f,t}(j)/P_{f,t})$
$R_{d,s,t}$	Domestic interest rate	$r_{d,s,t} = \ln (R_{d,s,t})$
$R_{f,s,t}$	Foreign interest rate	$r_{f,s,t} = \ln (R_{f,s,t})$
$R_{d,l,t}$	Domestic long-term interest rate	$r_{d,l,t} = \ln (R_{d,l,t})$
$R_{f,l,t}$	Foreign long-term interest rate	$r_{f,l,t} = \ln (R_{f,l,t})$
$W_{d,t}$	Domestic wage	$w_{d,t} = \ln (W_{d,t}/P_{d,t}Z_t)$
$W_{f,t}$	Foreign wage	$w_{f,t} = \ln (W_{f,t}/P_{f,t}Z_t)$
$Y_{d,t}$	Domestic demand	$y_{d,t} = \ln (Y_{d,t}/Z_t)$
$Y_{f,t}$	Foreign demand	$y_{f,t} = \ln (Y_{f,t}/Z_t)$
$Y_{F,d,t}$	Domestic GDP (production of domestic firms)	$y_{F,d,t} = \ln (Y_{F,d,t}/Z_t)$
$Y_{F,f,t}$	Foreign GDP (production of foreign firms)	$y_{F,f,t} = \ln (Y_{F,f,t}/Z_t)$
$\lambda_{d,t}$	Domestic household Lagrange multiplier	$\lambda_{d,t} = \lambda_{d,t}$
$\lambda_{f,t}$	Foreign household Lagrange multiplier	$\lambda_{f,t} = \lambda_{f,t}$

Table 1: Description of the variables and corresponding stationnary expression.

$\varepsilon^u_{d,t}$	Domestic households intertemporal preference shock	$\phi^u_{d,t} = \ln\left(\varepsilon^u_{d,t}/\varepsilon^u_{d,t-1}\right)$
$\varepsilon^u_{f,t}$	Foreign households intertemporal preference shock	$\phi^u_{f,t} = \ln\left(\varepsilon^u_{f,t}/\varepsilon^u_{f,t-1}\right)$
$\varepsilon^L_{d,t}$	Domestic households labor supply shock	$\phi^L_{d,t} = \ln\left(\varepsilon^L_{d,t}\right)$
$\varepsilon^L_{f,t}$	Foreign households labor supply shock	$\phi^L_{f,t} = \ln\left(\varepsilon^L_{f,t}\right)$
$\varepsilon^M_{d,t}$	Domestic households liquidity preference shock	$\phi^M_{d,t} = \ln\left(\varepsilon^M_{d,t}\right)$
$\varepsilon^M_{f,t}$	Foreign households liquidity preference shock	$\phi^M_{f,t} = \ln\left(\varepsilon^M_{f,t}\right)$
$\varepsilon^R_{d,t}$	Domestic monetary policy shock	$\phi^R_{d,t} = \varepsilon^R_{d,t}$
$\varepsilon^R_{f,t}$	Foreign monetary policy shock	$\phi^R_{f,t} = \varepsilon^R_{f,t}$
$\varepsilon^\theta_{d,t}$	Domestic demand elasticity shock	$\phi^\theta_{d,t} = \varepsilon^\theta_{d,t}$
$\varepsilon^\theta_{f,t}$	Foreign demand elasticity shock	$\phi^\theta_{f,t} = \varepsilon^\theta_{f,t}$
ε_t	Technology shock	$\phi^y_t = \ln\left(\varepsilon_t/\varepsilon_{t-1}\right)$

Table 2: Description of the shock variables and corresponding stationnary expression.

3 Summary of parameters

1. Parameters ($\overline{\eta}_{i,j}$; $\eta_{i,j}$; std of $\xi^j_{i,t}$) for each exogenous process $\phi^j_{i,t}$.

2. Monetary policy rule's parameters: $\gamma_{d,R}$; $\gamma_{d,P}$; $\gamma_{d,Y}$; $\gamma_{d,f}$; $\gamma_{f,R}$; $\gamma_{f,P}$; $\gamma_{f,Y}$; $\gamma_{f,f}$.

3. Bonds' rigidity's parameters: $\varphi_{d,s,d}$; $\varphi_{d,s,f}$; $\varphi_{d,l,d}$; $\varphi_{d,s,f}$; $\varphi_{f,s,d}$; $\varphi_{f,s,f}$; $\varphi_{f,l,d}$; $\varphi_{f,l,f}$; $\mu_{d,s,d}$; $\mu_{d,s,f}$; $\mu_{d,l,d}$; $\mu_{d,l,f}$; $\mu_{f,s,d}$; $\mu_{f,s,f}$; $\mu_{f,l,d}$; $\mu_{f,s,f}$;

4. Price rigidity's parameters: $\mu_{d,P}$; $\mu_{f,P}$.

5. Habits' parameters: $h_{d,C}$; $h_{f,C}$.

6. Long term bonds' parameters: s_d; s_f.

7. Indexation parameters: v_d; v_f.

8. Demand structure preferences's parameters: ω_D; ω_f.

References

Rotemberg, J. J., 1982a. Monopolistic price adjustment and aggregate output. Review of Economic Studies 49 (4), 517–31.

Rotemberg, J. J., 1982b. Sticky prices in the United States. Journal of Political Economy 90 (6), 1187–1211.