The Zero Bound on Interest Rates and Optimal Monetary Policy

G. Eggertsson and M. Woodford

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Lecture 1

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This presentation does not necessarily reflect the views of the Bank of Israel

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Literature review

- Japanese interest rate almost zero since 1996.
- Low output and price deflation.
- "*Vigorous* expansion of the monetary base has also seemed to do little to stimulate demand under these circumstances".
- When nominal interest rates are at zero, which tool can be used to stabilize the economy ? (liquidity trap, Keynes)
- Krugman (2002) see deflation as a "black hole" (i.e. an economy cannot expect to escape once it has entered).

Some questions

- What is the efficacy of monetary policy in a liquidity trap ?
- Assets purchases by central banks (including long term bonds in order to shift longer-term interest rates).
- A Taylor-type/inflation targeting monetary policy rule cannot prevent from falling into a deflationary spiral/liquidity trap/ZLB.
- Credibility question: how to justify inflation targeting policies once the economy reached the deflationary spiral/liquidity trap/ZLB ? (K. Okina, 1999)

Eggertsson and Woodford: what do they do ?

- Consequences of the zero lower bound on nominal interest rates for the optimal conduct of monetary policy.
- Explicitly intertemporal equilibrium framework of the monetary transmission mechanism.
- How the ZLB existence changes the character of optimal monetary policy ?
- Particular emphasis to the role of expectations regarding future policy in determining the severity of the distortions that result from hitting the ZLB.

Eggertsson and Woodford: what do they find ?

- They find that ZLB does represent an important constraint on what monetary stabilization policy can achieve.
- The expected future path of nominal interest rates matters: a commitment to keep nominal interest rates low for a longer period of time should stimulate aggregate demand
  - even when current interest rates cannot be lowered further.
  - even under the hypothesis that inflation expectations would remain unaffected.
- Expanding the monetary base through central bank purchases of a variety of types of assets does little (if anything to expand the set of feasible paths for inflation and real activity that are consistent with equilibrium under some (fully credible) policy commitment).
- ZLB can indeed be temporarily binding, and when it is, it inevitably results in lower welfare than could be achieved in the absence of such a constraint.

Is quantitative easing a separate policy instrument ?

- Monetary policy ceases to be an effective instrument to head off economic contraction in a "liquidity trap" (Keynes, 1936).
- Both the demand for money and the role of financial assets (including the monetary base) in private sector budget constraints are modeled.
- Key result: irrelevance proposition for open-market operations in a variety of types of assets that the central bank might acquire (under the assumption that the open-market operations do not change the expected future conduct of monetary or fiscal policy).

Assumptions

- No endogenous variations in the capital stock.
- Perfectly flexible wages.
- Monopolistic competition in goods markets.
- Sticky prices *à la* Calvo (1983) → deflation has real effects.
- Continuum of industries j employing an industry-specific type of labor, with its own wage.
- Following Sidrauski (1967), real balances are included in the utility function, as a proxy for the services that money balances provide in facilitating transactions (functional equivalence between CIA and MIU features).
- For simplicity: complete financial markets and no limit on borrowing against future income are assumed.
- Non-separability between consumption and real money.[2]

[2] As in Benchimol and Fourçans (JoM, 2012; MD, forthcoming).

Representative household

- Households seek to maximize:

$$E_t \sum_{k=0}^{+\infty} \beta^k \left\{ u\left(C_{t+k}, \frac{M_{t+k}}{P_{t+k}}; \xi_{t+k} \right) - \int_0^1 v\left(H_{t+k}\left(j\right); \xi_{t+k} \right) dj \right\} \tag{1}$$

where
- C_t is a Dixit-Stiglitz aggregate of consumption (EoS $\theta > 1$).
- P_t is a Dixit-Stiglitz price index.
- M_t / P_t represents real money balances.
- $H_t\left(j\right)$ is the quantity supplied of labor of type j.
- $\forall k$, u_t is a concave function, increasing in the first argument and increasing in the second for all levels of real balances up to a satiation level $\bar{m}\left(C_t; \xi_t\right)$, and v_t is an increasing convex function.

Intertemporal budget constraint

$$
\begin{aligned}
& E_t \sum_{k=0}^{+\infty} Q_{t,t+k} \left[P_{t+k} C_{t+k} + \delta_{t+k} M_{t+k} \right] \\
\leq \quad & W_t + E_t \sum_{k=0}^{+\infty} Q_{t,t+k} \left[\begin{array}{c} \int_0^1 \Pi_{t+k}(i)\, di - T_{t+k} \\ + \int_0^1 w_{t+k}(j) H_{t+k}(j)\, dj \end{array} \right]
\end{aligned} \tag{2}
$$

where

- $Q_{t,t+k} = \beta^k \frac{u_{c,t+k}}{u_{c,t}}$ is the SDF valuing random nominal income at date t in monetary units at date $t+k$.
- $\delta_t = i_t / (1 + i_t)$ is the opportunity cost of holding money.
- W_t is financial wealth (including nominal money holdings).
- $\Pi_t(i)$ represents the nominal profits of the supplier of good i.
- $w_t(j)$ is the nominal wage earned by labor of type j.
- T_t represents the net nominal tax liabilities of each household.

Solving the model

- Euler equation

$$\frac{1}{1+i_t} = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \right] \tag{3}$$

where $\lambda_t = u_{c,t}$ is the Lagrange multiplier.

- Optimal substitution between real money balances and expenditure

$$u_{m,t} = \lambda_t \delta_t \tag{4}$$

where it is assumed that
- zero interest is paid on the monetary base.
- preferences are such as to exclude the possibility of a corner solution with zero money balances.

- Simple equilibrium condition: $C_t = Y_t$.

- Assuming that both consumption and liquidity services are normal goods, a unique level of real balances, $L\left(Y_t, i_t; \xi_t\right)$, solves this equilibrium condition, such as

$$\frac{M_t}{P_t} \geq L\left(Y_t, i_t; \xi_t\right) \tag{5}$$

$$i_t \geq 0 \tag{6}$$

- Then, the minimum level of real balances for which $u_m = 0$ is

$$L\left(Y, 0; \xi\right) = \bar{m}\left(C; \xi\right) \tag{7}$$

- Budget constraint existence

$$\sum_{k=0}^{+\infty} \beta^k E_t \left[u_{c,t+k} Y_{t+k} + u_{m,t+k} \frac{M_{t+k}}{P_{t+k}}\right] < \infty \tag{8}$$

- Transversality condition

$$\lim_{k \to \infty} \beta^k \left[u_{c,t+k} \frac{D_{t+k}}{P_{t+k}} \right] = 0 \tag{9}$$

where D_t represents the total nominal value of government liabilities (monetary base plus government debt).

Conditions 3 to 9 suffice to imply that the representative household chooses optimal consumption and portfolio plans (including its planned holdings of money balances) given its income expectations and the prices (including financial asset prices) that it faces, while making choices that are consistent with financial market clearing.

- Each differentiated good i is supplied by a single, monopolistically competitive producer.
- There are assumed to be many goods in each of an infinite number of industries: each industry j uses a type of labor that is specific to that industry, and all goods in an industry change their prices at the same time.
- The representative household supplies all types of labor and consumes all types of goods.
- Each good is produced in accordance with a common production function such as

$$y_t(i) = A_t f\left[h_t(i)\right] \tag{10}$$

where

- A_t is an exogenous productivity factor common to all industries.
- $f(.)$ is an increasing, concave production function.
- $h_t(i)$ is the industry-specific labor hired by firm i.

- Nominal profits (sales revenue in excess of labor costs) are

$$profits_t = p_t\left(i\right)Y_t\left[\frac{p_t\left(i\right)}{P_t}\right]^{-\theta} - \frac{v_{h,t}}{u_{c,t}}P_t f^{-1}\left\{\frac{Y_t\left[\frac{p_t(i)}{P_t}\right]^{-\theta}}{A_t}\right\} \quad (11)$$

- If prices were fully flexible, $p_t\left(i\right)$ would be chosen each period to maximize this function.
- Calvo (1983) prices are defined as usually (previous lectures).

- Central bank's short-term nominal interest rate is determined by a feedback rule in the spirit of the Taylor rule such as

$$i_t = \Phi_t \left(\pi_t, Y_t; \tilde{\xi}_t\right) \geq 0 \tag{12}$$

where $\tilde{\xi}_t = \xi_t +$ exogenous disturbances.

- Such a rule implies that the central bank supplies the quantity of base money that happens to be demanded at the interest rate given by this formula.
- Hence, equation 12 implies a path for the monetary base, so long as the value of Φ is (strictly) positive.
- If $\Phi = 0$, the policy commitment in equation 12 implies only a lower bound on the monetary base that must be supplied.
- Then, greater or smaller base money supply is questionable.

Base-supply rule and QE

- A base-supply rule is assumed such as

$$\frac{M_t}{P_t} = \Psi L\left(Y_t, \Phi_t; \xi_t\right) \tag{13}$$

where $\begin{cases} \Psi\left(\pi_t, Y_t; \tilde{\xi}_t\right) = 1 \text{ if } \Phi_t > 0 \\ \Psi\left(\pi_t, Y_t; \tilde{\xi}_t\right) \geq 1 \text{ if } \Phi_t \leq 0 \end{cases}$

- Such a base-supply rule is consistent with both the interest rate operating target specified in equation 12 and the equilibrium relations in expressions 5 and 6.
- QE as a policy tool can then be represented by Ψ.

Central bank's securities

- Vector of central bank portfolio shares: ω_t^m.

Shares determined by a policy rule:

$$\omega_t^m = \omega_t^m \left(\pi_t, Y_t; \tilde{\xi}_t \right) \tag{14}$$

- Vector of nominal values of central bank holdings of the various securities: $M_t \omega_t^m$.
- Vector of asset holdings, z_t, and payoffs on these securities in each state of the world, specified by exogenously given (state-contingent) vectors a_t and b_t and matrix F_t, lead to delivery in period t of a quantity $a_t^T z_{t-1}$ of money, a quantity $b_t^T z_{t-1}$ of the consumption good and a vector $F_t^T z_{t-1}$ of securities.
- q_t is the vector of nominal asset prices in (ex-dividend) period-t trading.
- See note 20 of the paper for a detailed explanation.

Central bank's asset pricing

- Then, the gross nominal return on the jth asset between periods $t-1$ and t is then given by

$$R_t(j) = \frac{a_t(j) + P_t b_t(j) + q_t^T F_t(.,j)}{q_{t-1}(j)} \tag{15}$$

- Absence of arbitrage opportunities leads to

$$q_t^T = \sum_{k=1}^{+\infty} E_t Q_{t,t+k}\left[a_{t+k}^T + P_t b_{t+k}^T\right] \prod_{s=1}^{t+k-1} F_{t+s} \tag{16}$$

- Under the assumption that no interest is paid on the monetary base, the nominal transfer by the central bank to the public treasury each period is equal to

$$T_t^{cb} = R_t^T \omega_{t-1}^m M_{t-1} - M_{t-1} \tag{17}$$

Fiscal policy rules

- The acceptable level of real government liabilities is defined as a function of the preexisting level and various aspects of current macroeconomic conditions such as

$$\frac{D_t}{P_t} = d\left(\frac{D_{t-1}}{P_{t-1}}, \pi_t, Y_t; \tilde{\xi}_t\right) \tag{18}$$

where D_t is also used in the transversality condition (equation 9).

- The flow government budget constraint is now

$$D_t = R_t^T \omega_{t-1}^f B_{t-1} - T_t^{cb} - T_t^h \tag{19}$$

where

- $B_t = D_t - M_t$ is the total nominal value of end-of-period nonmonetary liabilities.
- T_t^h is the nominal value of the primary budget surplus.

Debt management policy

- Composition of the government's nonmonetary liabilities at each point in time is

$$\omega_t^f = \omega_t^f \left(\pi_t, Y_t; \tilde{\xi}_t\right) \tag{20}$$

Rational expectation equilibrium

- Aggregate demand block : Eq. 1 to Eq. 9.
- Aggregate supply block : Eq. 7 and 9 of the paper.
- Asset-pricing block: Eq. 16.
- Monetary policy block: Eq. 12 to Eq. 14.
- Fiscal policy block: Eq. 18 and Eq. 20.
- Then, we can define a rational expectations equilibrium, as a collection of stochastic processes, satisfying previous blocks

$$\left\{ p_t^*, P_t, Y_t, i_t, q_t, M_t, \omega_t^m, D_t, \omega_t^f \right\} \tag{21}$$

where $\{p_t^*, P_t, Y_t, i_t, q_t, D_t\}$ is independent of M_t (i.e. Ψ), ω_t^m, and ω_t^f (demonstration pages 157 to 159).

Demonstration

- Strong assumption: additional money balances beyond the satiation level, $\bar{m}$, provide no further liquidity services.
- Then, by differentiating this relation, we see further that $\frac{\partial u(C,m;\xi)}{\partial C}$ does not depend on the exact value of m either, as long as $m \geq \bar{m}$.

About quantitative easing

- Neither the extent to which quantitative easing is employed when the zero bound binds, nor the nature of the assets that the central bank may purchase through open-market operations, has any effect on whether a deflationary price-level path represents a rational expectations equilibrium.
- The EW's general-equilibrium analysis does not support the notion that expansions of the monetary base represent an additional tool of policy, independent of the specification of the rule for adjusting short-term nominal interest rates.
- Policy should matter in practice only insofar as they help to signal the nature of policy commitments (to the public).

About assumptions

- EW neglect portfolio-balance effects, which play an important role in the policy options that would remain available around ZLB.
- A central bank should be able to lower longer-term interest rates even when overnight rates are already at zero, through purchases of longer-maturity government bonds.
- This would shift the composition of the public debt in the hands of the public in a way that affects the term structure of interest rates.
- EW's framework applies only given a correct private sector understanding of the central bank's commitments regarding future policy.

How Severe a Constraint Is the Zero Bound?

- ZLB limits the set of possible equilibrium paths for prices and output, although the quantitative importance of this constraint remains to be seen.
- Management of expectations is the key to successful monetary policy at all times, not just in those relatively unusual circumstances.

- In the zero-inflation steady state, real rate of interest is $\bar{r} = \beta^{-1} - 1 > 0$.
- $\bar{r}$ is also the steady state nominal interest rate.
- Hence, in the case of small enough disturbances, optimal policy will involve a nominal interest rate that is always positive, and the zero bound will not be a binding constraint.
- They assume that interest paid on the monetary base, $i^m \geq 0$, cannot be reduced (for some institutional reason). Strong hypothesis ?
- Then the LB on interest rates becomes

$$i_t > i^m \tag{22}$$

- What is optimal policy ? Both amplitude of disturbances ($\|\xi\|$) and steady-state opportunity cost of holding money ($\delta = \frac{\bar{r}-i^m}{1+\bar{r}} > 0$) are small enough.

Reminder

$$x_t = E_t[x_{t+1}] - \sigma(i_t - E_t[\pi_{t+1}] - r_t^n) \quad (23)$$

$$\pi_t = \kappa x_t + \beta E_t[\pi_{t+1}] + u_t \quad (24)$$

where $\pi_t = \ln(P_t / P_{t-1})$ is the inflation rate, x_t is a welfare-relevant output gap, and i_t is now the continuously compounded nominal interest rate, corresponding to $\ln(1 + i_t)$ in the notation used previously. u_t is a cost-push disturbance and r_t^n is the equilibrium real rate of interest.

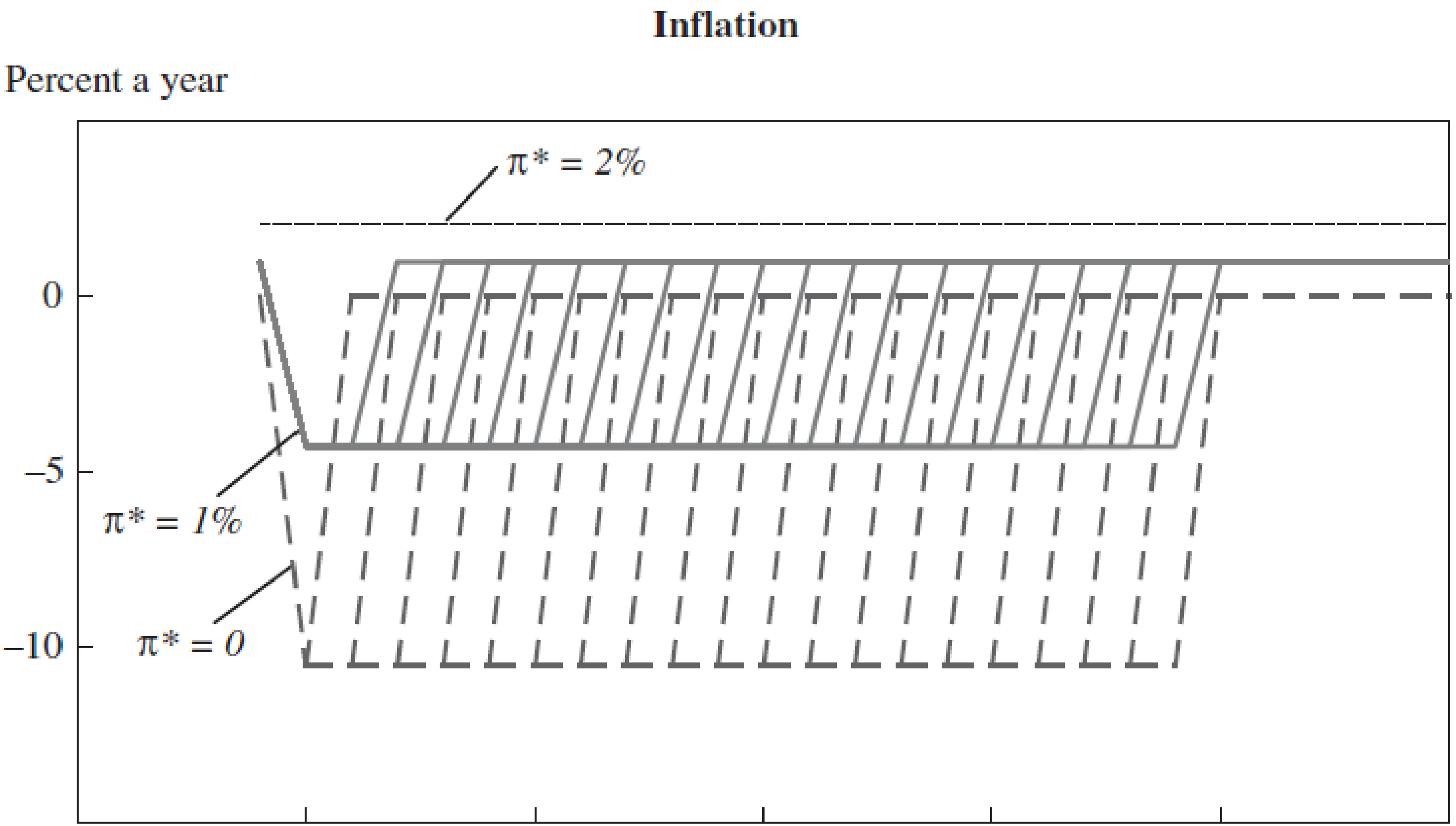
ZLB

A necessary condition for satisfying inflation targeting $(\pi_t = \pi^*)$ is

$$i_t = r_t^n + \pi^* \tag{25}$$

- Indeed, when inflation is on target, the real interest rate is equal to the natural real rate at all times, and the output gap is at its long-run level.
- The zero bound, however, prevents equation 25 from holding if $r_t^n < -\pi^*$.
- Thus, if the natural rate of interest is low, the zero bound frustrates the central bank's ability to implement an inflation target.

Figure 2. State-Contingent Responses of Inflation and the Output Gap to a Shock to the Natural Rate of Interest under Strict Inflation Targeting[a]



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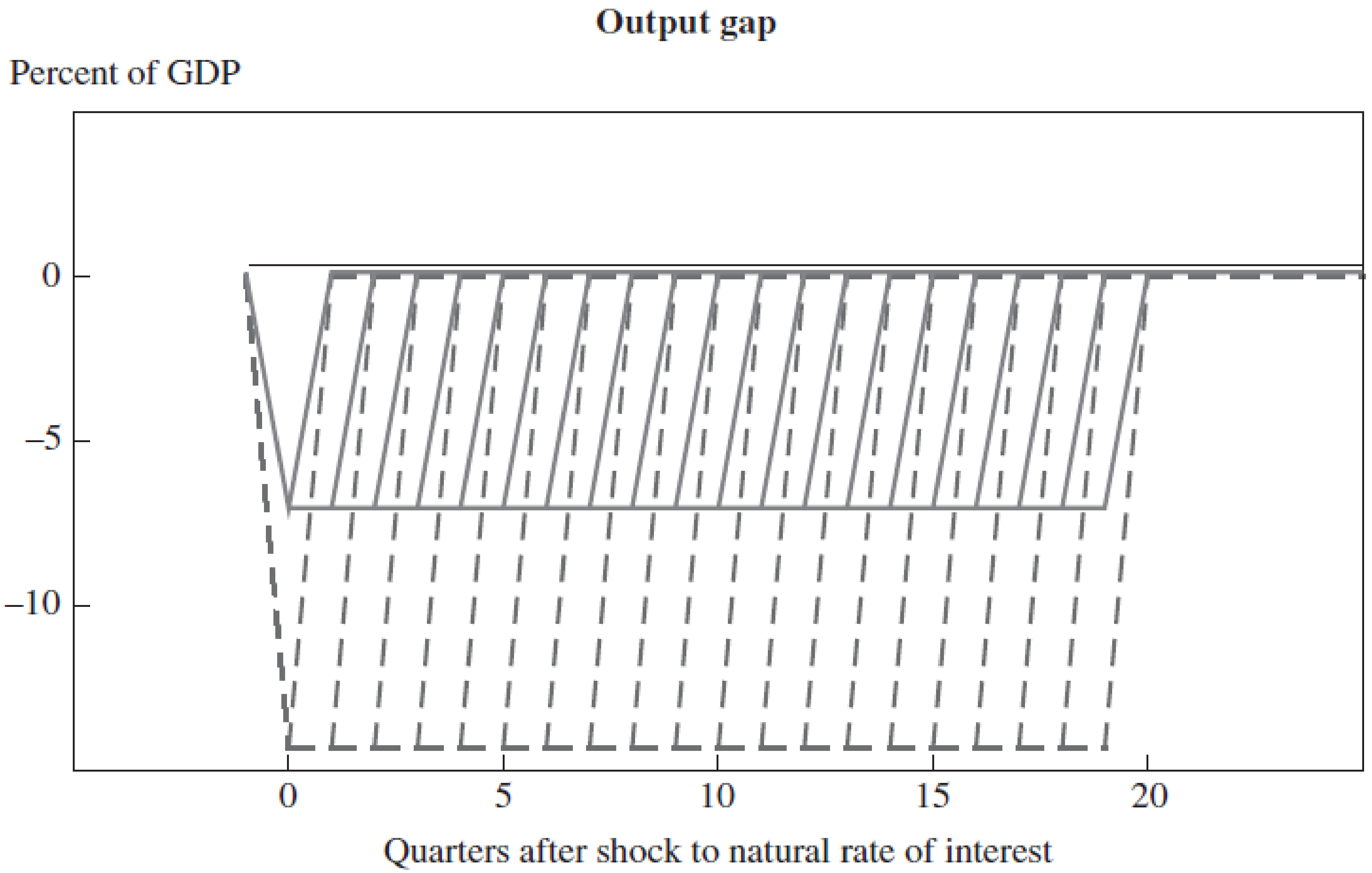


- See and comment Figure 2 (page 171).
- Suppose the natural rate of interest is unexpectedly negative in period 0 and reverts back to its steady-state value $\bar{r} > 0$ with a fixed probability in every period.
- Figure 2 shows the state-contingent paths of the output gap and inflation under these circumstances for each of three different possible inflation targets π^*.
- We assume in period 0 that the natural rate of interest becomes -2% a year and then reverts back to the steady-state value of $+4\%$ a year with a probability of 10% each quarter.
- Thus the natural rate of interest is expected to be negative for ten quarters on average at the time the shock occurs.
- The dashed lines in figure 2 show the state-contingent paths of the output gap and inflation if the central bank targets zero inflation.

Optimal monetary policy under commitment

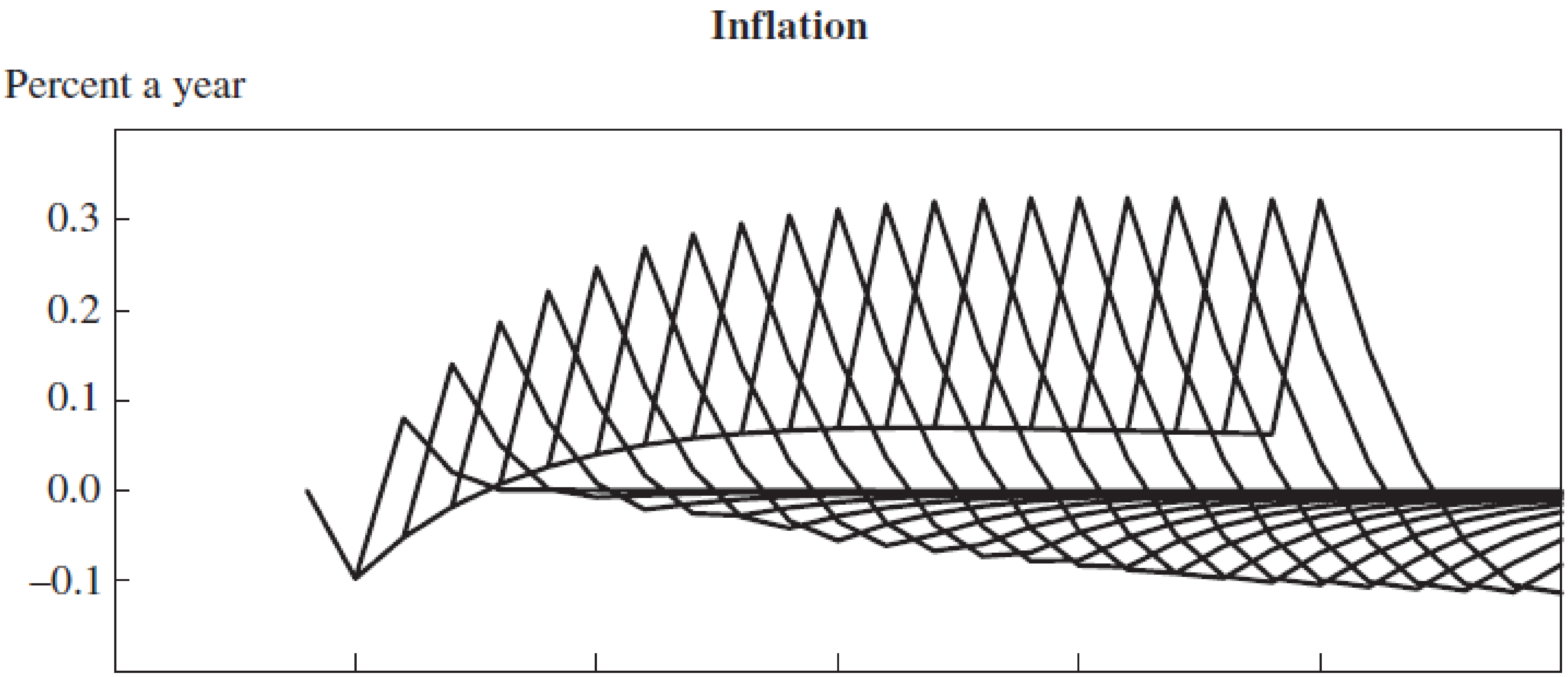
$$\min\left\{E_0\left[\sum_{k=0}^{+\infty}\beta^k\left(\pi_k^2+\lambda x_k^2\right)\right]\right\} \tag{26}$$

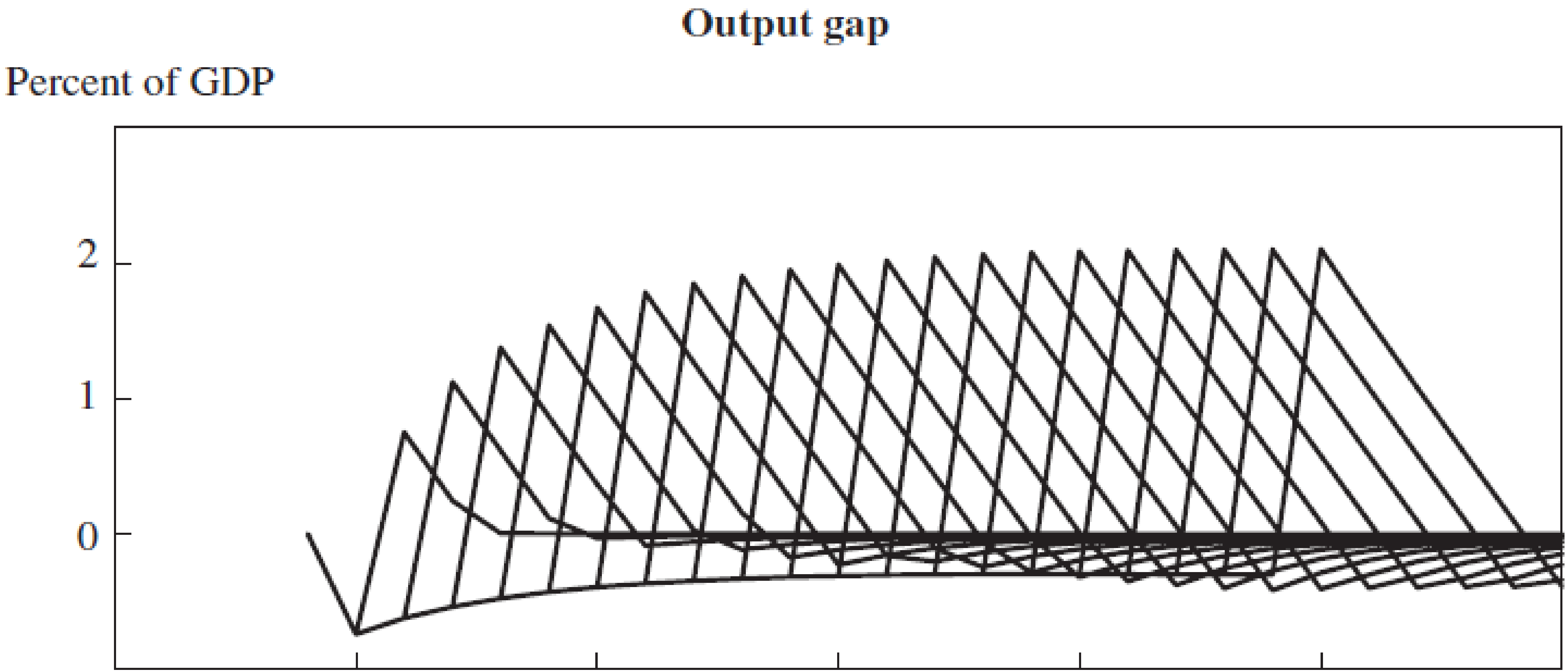
- At the ZLB, Lagrangian (under constraints Eq. 23 and Eq. 24) is

$$L_0=E_0\left\{\sum_{k=0}^{+\infty}\beta^k\left(\begin{array}{c}\pi_k^2+\lambda x_k^2\\ +\varphi_{1k}\left(x_k-x_{k+1}-\sigma\left(\pi_{k+1}+r_k^n\right)\right)\\ +\varphi_{2k}\left(\pi_k-\kappa x_k-\beta\pi_{k+1}-u_k\right)\end{array}\right)\right\} \tag{27}$$

- Impossible to formally solve because of the nonlinear constraint $\varphi_{1t}i_t=0$ (numerical solution in the paper).
- FOC $\longrightarrow$ optimal policy is history dependent $\longrightarrow$ optimal choice of inflation, the output gap, and the nominal interest rate depends on the past values of the endogenous variables (lagged Lagrangian multiplier).

Figure 3. Inflation, the Output Gap, and Prices under the Optimal Policy Commitment[a]




Output gap
Percent of GDP
2
1
0

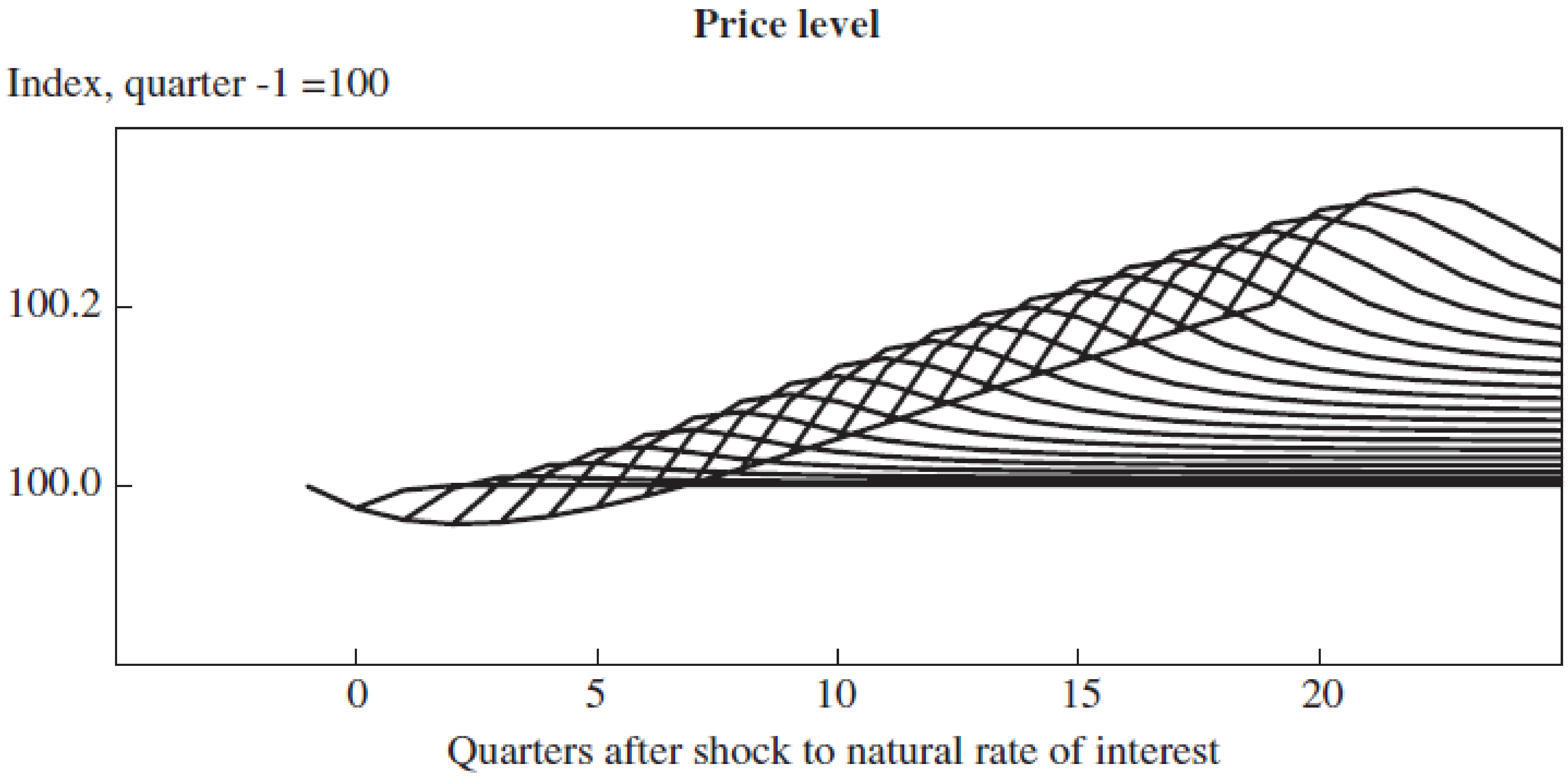
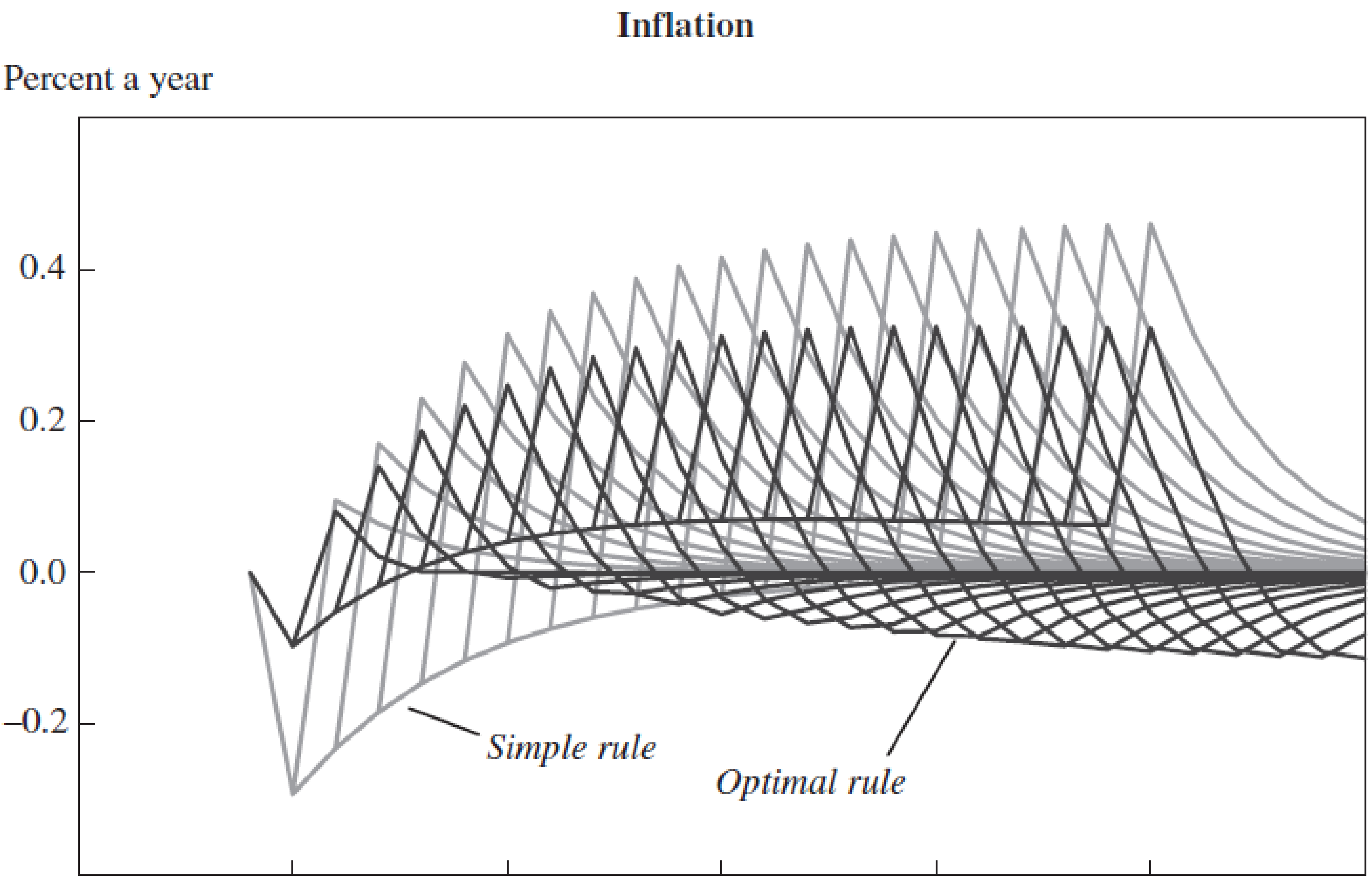
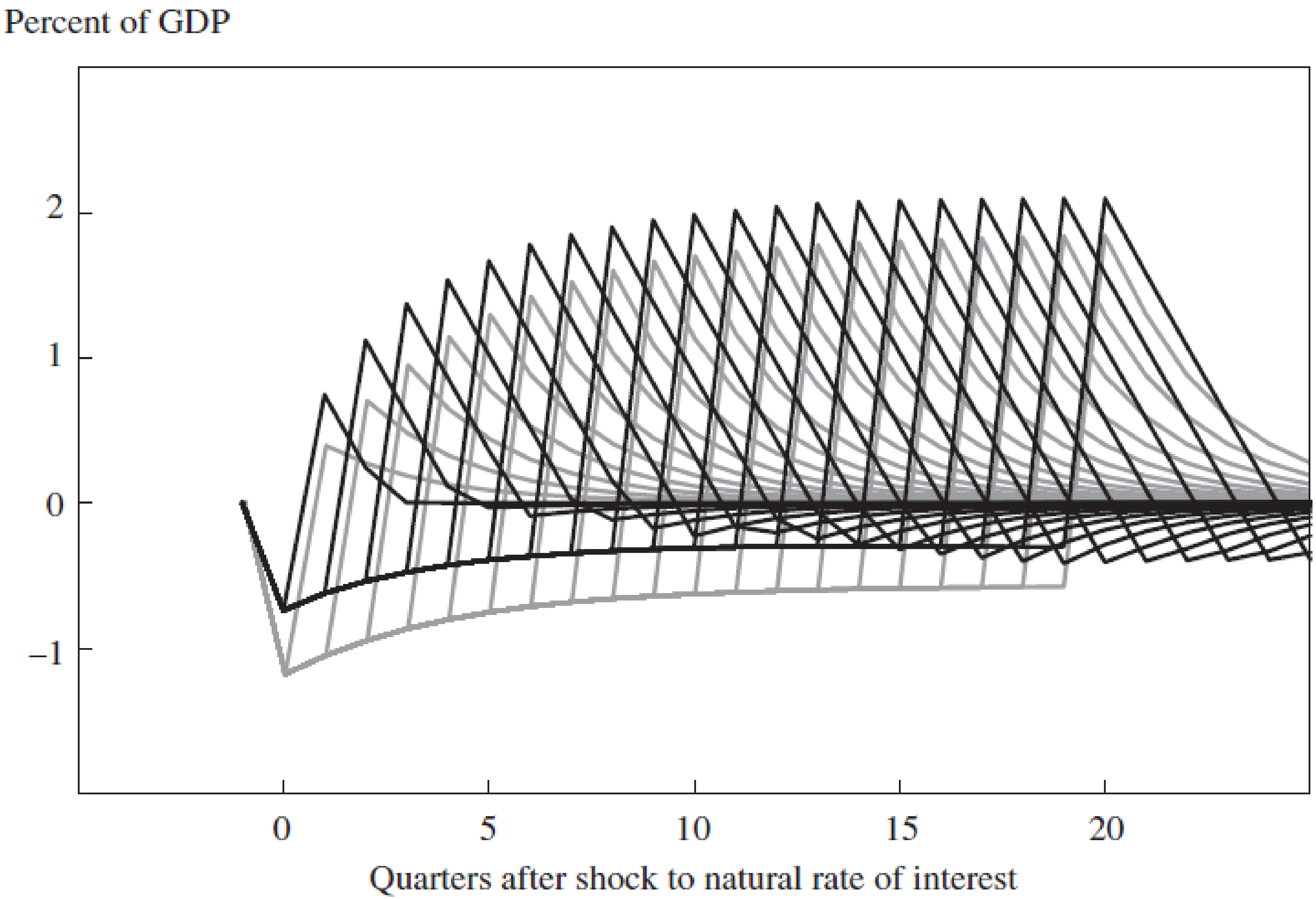

Price level
Index, quarter -1 =100
100.2
100.0
0
5
10
15
20
Quarters after shock to natural rate of interest

Figure 7. Responses of Inflation and the Output Gap under the Optimal Targeting Rule and under the Simple Rule[a]



Output gap


Percent of GDP
2
1
0
−1
0
5
10
15
20
Quarters after shock to natural rate of interest

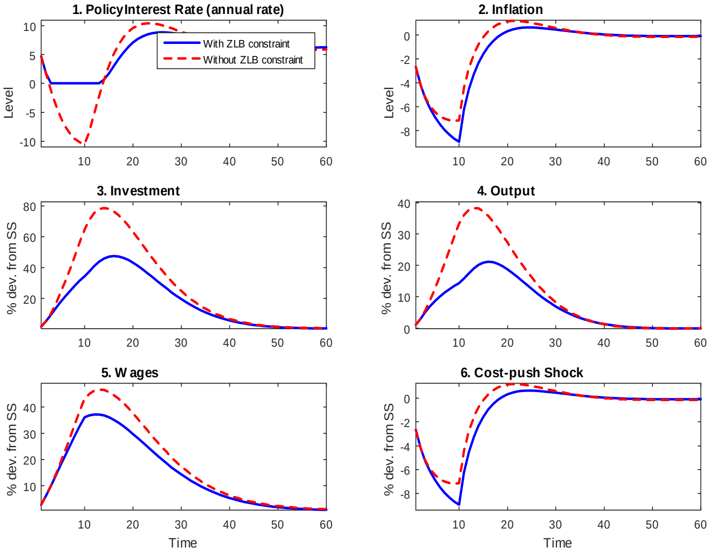
- See and comment Figure 3 (page 177).
- Figure 3 shows the optimal output gap, the inflation rate, and the price level from period 0 to period 25.
- As in figure 2, the separate lines in each panel show the evolution of the variables in the case that the disturbances last for different lengths of time ranging from one quarter to twenty quarters.
- Optimal policy involves committing to the creation of an output boom once the natural rate again becomes positive $\longrightarrow$ creation of future inflation.
- Such a commitment stimulates aggregate demand and reduces deflationary pressure while the economy remains in the liquidity trap, through each of several channels.
- EW tests several monetary policy rules under several targets and how they respond to shocks (see figures 4 to 9)

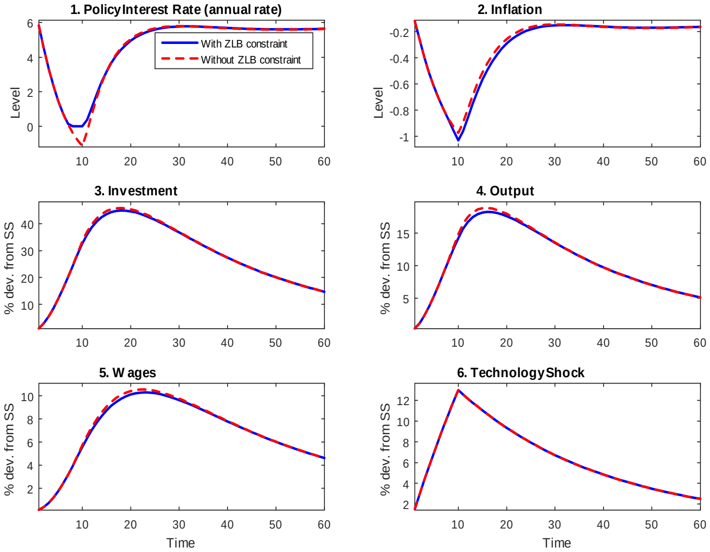
- EW conclude that price-level targeting rules, which are history-dependent policies, are vastly superior to any of the strict inflation targets.
- Indeed, a price-level target commits the government to undo any deflation with subsequent inflation.
- Larger disturbance $\rightarrow$ larger initial deflation $\rightarrow$ greater inflation expectations.
- An automatic stabilizer is built into the price-level target, which is lacking under a strict inflation targeting regime.
- EW show that a proper strategy for the central bank to use in communicating its objectives and targets when outside the liquidity trap is of crucial importance for this policy rule to be successful.
- EW also show that a commitment of no government Ponzi game (that is the government will asymptotically be neither creditor nor debtor) exclude the possibility of a self-fulfilling deflationary trap.

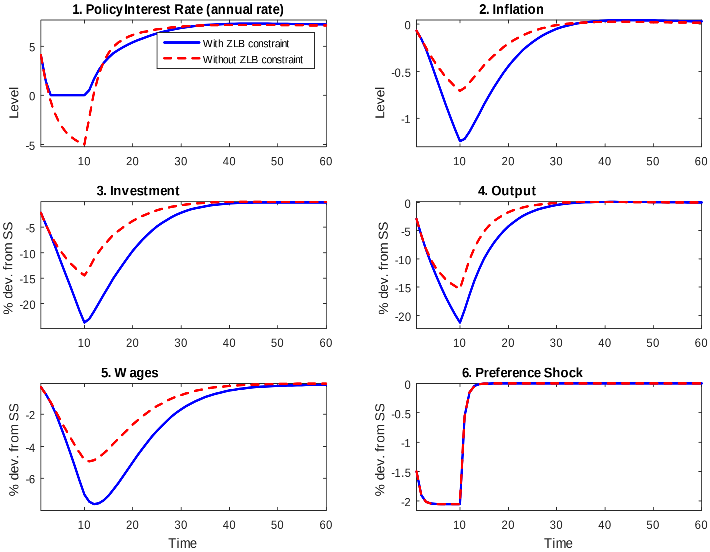
- Key to deal with a situation in which monetary policy is constrained by the ZLB is the skillful management of expectations regarding the future conduct of policy (forward guidance).
- In other words, a central bank should be able to commit itself in advance to a course of action that is desirable because of the benefits that flow from its being anticipated.
- And then to work to make that commitment credible to the private sector.
- A simple purely forward-looking approach to policy can lead to quite bad outcomes.
- Optimal policy, under the assumption that credible commitment is possible, leads to a better outcome.
- The central bank should clearly understand the kind of history-dependent behavior to which it should be seen to be committed.
- Conducting policy in accordance with a rule may not suffice.

ZLB and SW model

- We use the Smets and Wouters (2007) model calibrated as in their AER paper.
- We simulate the model with and without the ZLB constraint on the nominal interest rate in order to understand the importance of such a constraint.
- We focus on 3 shocks: a cost-push shock, a technology shock, and a preference shock.


1. PolicyInterest Rate (annual rate)
Level
With ZLB constraint
Without ZLB constraint
2. Inflation
Level
3. Investment
% dev. from SS
4. Output
% dev. from SS
5. W ages
% dev. from SS
Time
6. Cost-push Shock
% dev. from SS
Time


1. Policy Interest Rate (annual rate)
Level
With ZLB constraint
Without ZLB constraint
2. Inflation
Level
3. Investment
% dev. from SS
4. Output
% dev. from SS
5. Wages
% dev. from SS
6. Technology Shock
% dev. from SS
Time
Time


1. Policy Interest Rate (annual rate)
Level
With ZLB constraint
Without ZLB constraint
2. Inflation
Level
3. Investment
% dev. from SS
4. Output
% dev. from SS
5. Wages
% dev. from SS
6. Preference Shock
% dev. from SS
Time
Time