# Wealth Effects and Monetary Policy in the Euro Area

Jonathan Benchimol*

December 2007

*Preleminary version, do not quote.*

**Abstract**

We study the impact of wealth effects on output and inflation dynamics in an heterogeneous monetary area. We show how wealth effects and heterogeneities modify the central bank's reaction function and some transmission mechanisms. By building a basic model incorporating wealth effects, we compute a one-country optimal taylor rule. With this basic model and by incorporating heterogeneities, we compute a two-country optimal Taylor rule. We show, ceteris paribus, increasing output heterogeneity involves a restrictive monetary policy and increasing inflation heterogeneity involves an accommodating monetary policy. We show increasing wealth effect weight on demand function in one of the two country involve an accommodating monetary policy.

*Keywords:* ECB, monetary policy, wealth effects, taylor rules.
*JEL Classification:* E44, E52, F41.

## 1 Introduction

During the 2007's crisis, subprime mortgage loans increasingly fell into default, triggering downgrades of investment securities backed by pools of these loans, and a lot of central bankers have realized wealth effects were more important than expected. Many investors did not foresee the risk of collateralized debt securities. In response to this crisis, the Fed has been trying to keep a steady hand and prevent a credit crunch. We don't yet know how well the Fed has succeeded, or how well it could have done in the first place. And the storm has not yet fully passed. Changes in house prices could have a bigger effect on consumption than the traditional "wealth effect" suggest, Ben Bernanke (June 2007) said in comments that offer some insight into how the Federal Reserve may think about the continuing problems in the US housing market.

*ESSEC Business School, Department of Economics, Avenue Bernard Hirsch 95021 Cergy Pontoise Cedex 2, France, and University Paris 1 Panthéon-Sorbonne, CES, 106-112 Boulevard de l'Hôpital, 75647 Paris Cedex 13, France (e-mail: benchimol@essec.fr).

Central bank consideration of such an effect in its Taylor-type reaction rule is still complex. Svensson (1996) modeled the central bank reaction rule with inflation targeting. Targeting inflation has almost become the central banks norm of industrialized countries. Svensson (1997) showed that the targeting rules commitment is better than instruments rules commitment, and used a model incorporating in its system (accelerationist Phillips curve and demand curve) an exogenous variable. We resume this model to build our system of equations modeling the economy by incorporating the financial wealth effect (or property wealth effect). Following Vickers (1999), asset prices could be a part of monetary policy objectives and/or a part of the information used to follow these objectives. For Bernanke and Gertler (2001), asset prices volatility affect monetary policy only if it affect central bank inflation forecasts. Cecchetti (2000) suggest that the central bank can reduce long term inflation and output volatility by adjusting interest rates in response of asset prices misalignments. Filardo (2004) shows that if monetary policy ignores asset prices inflation, consumption and output prices inflation become less stable. Others believe that asset prices are too erratic in order to use the information contained, and any attempt to introduce an explicit reaction rule could deteriorate the economy. Moreover, Bernanke and Gertler (1999) have modeled it for non-determinists speculative bubbles processes and suggest that a too aggressive interest rate response increases inflation volatility, thus destabilizing the economy.

Yet Goodhart (2001) shows that the asset prices movements are associated with general trends of inflation. Goodhart and Hoffman (2000) derived financial conditions indexes using assets such as shares or property in their calculations and shown that it could be useful indicator for future consumer prices inflation. Backstrom (2000) argues that in a regime which explicit inflation targets clearly, asset prices are taking into account in aggregate demand. The link between aggregate demand and asset prices is the starting point of our model in order to model the wealth effect transmission channel. The aim of this paper is to model theoretically wealth effect and its impact on inflation and output in a heterogeneous financial area (and homogeneous monetary area). After presenting the basic one-country model and its resolution, solution of the central banker optimization problem is calculated in order to define the optimal interest rate rule. We presents the interpretation of results and we enhance the basic model for two countries with output, inflation and wealth effects heterogeneity.

## 2 The one-country model

We use a structural model of the economy allows us to taking into account, in the aggregate demand, asset prices changes. The first equation is a classical equation in the literature. The second is from Svensson (1997), adding an exogenous variable in the demand equation. The third equation is a classic expression of wealth effects as a function of real interest rate and expected demand at $t$ to $t+1$. We replace the Svensson's exogenous variable by a wealth effects variable, $w_t$, only in the demand function.

The first equation is a standard accelerationist (or backward-looking NAIRU type) Phillips Curve where the change in inflation is a positive function of the lagged output gap and the inflation shock. Such a specification has also been adopted by Ball (1999), Svensson (1997) and Rudebusch and Svensson (1999). Furher (1997) has also proved with American data that the forward looking is not significantly different for this equation:

$$\pi_{t+1} = \pi_t + \alpha_1 y_t + \varepsilon_{t+1} \tag{1}$$

where $\pi_t = p_t - p_{t-1} - \Delta p^*$ where $p_t$ is the logarithm of the price level at $t$, $\Delta p^*$ is the inflation target level and $\alpha_1$ is a parameter. $y_t$ is the output log-deviation from its equilibrium level (output gap). $\varepsilon_{t+1}$ is an error term representing shocks to inflation.

The second equation is a classical demand function, used by Walsh (1998), Ball (1997) and Svensson (1997) and enhanced by wealth effects (asset prices as index of financial/property market):

$$y_{t+1} = \beta_1 y_t - \beta_2 \left(i_t - E_t\left[\pi_{t+1}\right]\right) + \beta_3 w_t + \eta_{t+1} \tag{2}$$

where $i_t$ is the central bank (*repo*) interest rate instrument at $t$, chosen by the central banker. $0 \leq \beta_1 < 1$, $\beta_2 > 0$ and $\beta_3 \geq 1$ are parameters. In the literature, we have $\beta_2 = \frac{1}{\sigma}$ (intertemporal elasticity of substitution). $E_t\left[\pi_{t+1}\right]$ is the expectation operator conditional on the central bank's information set at time $t$. $\eta_{t+1}$ is an error term representing shocks to output.

The third equation matches with effective market returns (not only dividends). It is the log-difference between asset prices (as index) at the beginning and the end of the period as:

$$w_t = -\gamma_1 \left(i_t - E_t\left[\pi_{t+1}\right]\right) + \gamma_2 E_t\left[y_{t+1}\right] + u_t \tag{3}$$

where $w_t$ is the real wealth effect (in log-deviation) of asset prices at $t$, $\gamma_1 > 0$ and $\gamma_2 > 0$ are parameters. According to most empirical studies examining macroeconomic variables impact on the financial market (Fama (1981), Conover, Jensen and Johnson (1999)), assets return is negatively related to real interest rate and positively associated with output expectation. $u_t$ is an error term representing shocks to wealth effect.

$\varepsilon_{t+1}$, $\eta_{t+1}$ and $u_t$ represent exogenous random shocks respectively to inflation, aggregate demand, and asset price fundamentals. We assume that they are mutually uncorrelated *i.i.d.* processes with zero means and constant variances. This assumption is maybe strong but sufficiently realistic for our framework.

## 2.1 Resolution of the model

Starting from the previous equations, we obtain an expression of $E_t[y_{t+1}]$ depending on the model variables[1] , i.e. output, $y_t$ and real interest rate, $(i_t - \pi_t)$:

$$E_t[y_{t+1}] = \delta_1 y_t + \delta_2 \left(i_t - \pi_t\right) \tag{4}$$

[1] See Appendix 1 for more details, and a definition of $\delta_1$ and $\delta_2$.

where $\delta_1$ and $\delta_1$ depend on parameters. We replace this expression of $E_t[y_{t+1}]$ in the equation (3) and the expression of $E_t[\pi_{t+1}]$ in equation (2). We obtain equations of the following form:

$$\begin{aligned} \pi_{t+1} &= \kappa_t + \varepsilon_{t+1} \\ y_{t+1} &= \varphi_t + \nu_{t+1} \end{aligned} \tag{5}$$

where $\varphi_t = Ay_t - B(i_t - \pi_t)$ and $\kappa_t = \pi_t + \alpha_1 y_t$. $A$ and $B$ depend on parameters[2].

Following Walsh (1998), $\pi_t$ and $y_t$ are predetermined when we choose $i_t$. Thus, we can define $\varphi_t$ as the central bank's control variable and $\kappa_t$ as the state variable of the system.

## 2.2 Optimal interest rate rule

The main central bank's objective is to solve the following dynamic (intertemporal) optimization problem:

$$\min_{\varphi_t} \left\{ \frac{1}{2} E_t \left[ \sum_{i=1}^{\infty} \beta^i \left( (\kappa_t + \varepsilon_{t+1})^2 + \mu (\varphi_t + \nu_{t+1})^2 \right) \right] \right\} \tag{6}$$

under the following constraint: $\kappa_{t+1} = \pi_{t+1} + \alpha_1 y_{t+1} = \kappa_t + \alpha_1 \varphi_t + \omega_{t+1}$.

Where $\mu$ is the relative central bank's weight on output stabilization, $\beta$ is the policy discount factor, assumed constant for simplicity such as $0 < \beta < 1$ , and $\omega_{t+1} = \varepsilon_{t+1} + \alpha_1 \nu_{t+1}$ is an exogenous random shock of the state variable $\kappa_{t+1}$. At $t$, when $i_t$ is determined, $\varphi_t$ is determined and so the only state variable is $\kappa_t$. The value function is determined by the state variable $\kappa_t$. By applying the Bellman principle (dynamic programming), yields:

$$V(\kappa_t) = \min_{\varphi_t} E_t \left\{ \frac{1}{2} \left[ (\kappa_t + \varepsilon_{t+1})^2 + \mu (\varphi_t + \nu_{t+1})^2 \right] + \beta V(\kappa_t + \alpha_1 \varphi_t + \omega_{t+1}) \right\} \tag{7}$$

We solve the optimization program in order to find the optimal reaction rule[3]. Finally, we obtain the optimal interest rate knowing that $\varphi_t = Ay_t - B(i_t - \pi_t)$ and $\varphi_t = c\kappa_t$ where $c$ is defined in appendix 2. The resolution of this system leads us to the central bank's optimal reaction rule of the considered country, similarly to a classic Taylor rule[4] : $i_t = \lambda_\pi \pi_t + \lambda_y y_t$. The nominal interest rate is a linear function of inflation targeting and output gap.

## 2.3 Interpretation of results

In the standard case, without wealth effects, i.e. $\beta_3 = 0$ in equation 2, the system corresponds to a basic AS/AD model. In this standard case, optimal coefficients for inflation (targeting) and for output (gap) would be the following:

[2] See Appendix 1 for more details, and a defition of $A$ and $B$.
[3] See Appendix 2.
[4] See Appendix 3.

$\lambda_\pi^s = 1 - \frac{c}{\beta_2}$ and $\lambda_y^s = \alpha_1 + \frac{\beta_1 - \alpha_1 c}{\beta_2}$ because $c < 0$. With wealth effects, optimal coefficients become: $\lambda_\pi = 1 - \frac{c(1-\beta_3\gamma_2)}{\beta_2+\beta_3\gamma_1}$ and $\lambda_y = \alpha_1 + \frac{\beta_1 - \alpha_1 c(1-\beta_3\gamma_2)}{\beta_2+\beta_3\gamma_1}$. The inflation coefficient of the optimal reaction function should be stronger in the standard case than in the enhanced model with wealth effects consideration[5], i.e. $\lambda_\pi < \lambda_\pi^s$. Similarly, the output (gap) coefficient of the optimal reaction function should be stronger in the standard case than in the case with wealth effects[6], i.e. $\lambda_y < \lambda_y^s$. Accordingly, the consideration of wealth effects makes monetary policy more accommodative. In addition, the standard case is fully consistent with Taylor (1991): $\lambda_\pi^s > 1$ and $\lambda_y^s > 0$. Indeed, the standard inflation coefficient is greater than unity and the standard output gap coefficient is strictly positive. We compute partial derivatives of these two coefficients (inflation and output) with respect to parameters of the model[7].

**Proposition 1 (1)** *The stronger is the asset prices elasticity with respect to real interest rate, .i.e.* $\gamma_1$ *increases, and the more the central bank is accommodative (if* $\beta_3\gamma_2 < 1$*).*

We can interpret this like a double effect of the central bank: when the interest rates increase, there is a negative impact on corporate investments levels and there is a demoralization of the consumption from the decrease of effective (and/or expected) returns, what leads to a decrease of aggregate demand and inflation. We can also interpret it in terms of information: if wealth effects attach more importance (or related) to macroeconomic factors such as the real interest rate (incorporating, in our model, the nominal interest rate and the inflation rate), which is determined partly by the monetary authority, and more these information are taking into account by this monetary authority, in order to don't destabilize wealth effects and stabilize the economy.

Finally, we can simply interpret this result as a consequence of negative wealth effect created by interest rates rising. In this case, if a wealth effect decline occurs, i.e. an increase of $\gamma_1$, then the inflation and output gap coefficients decrease, and so the central bank is accommodative. Maybe this mechanism could have avoided the summer 2007's crisis. All these conclusions are true only if $\beta_3\gamma_2 < 1$, i.e. if wealth effect is inferior to the reverse of the asset prices elasticity with respect to expected output gap from the actual period for the period ahead.

**Proposition 2 (2)** *The stronger is the asset prices response to output gap expectation, i.e.* $\gamma_2$ *increases, and the more the central bank is accommodative.*

Growth is associated with an asset prices increase leading to an output increase what leads to a stronger central bank response to financial cycles in order to stabilize output. One can also remind that asset prices rising (at least national) may be due to growth increasing. However, in terms of information

[5] See Appendix 4.
[6] See Appendix 5.
[7] See Appendix 6.

used by the monetary authority, if output gap expectation is increasingly used in wealth effect, the demand function also contains this information and thus the demand function increases. This effect is offset by a joint decrease of the inflation and output gap coefficient of the central bank's optimal rule, in order to obtain a consistent reaction with macroeconomics reality.

**Proposition 3 (3)** *The stronger is the wealth effect (and impacts demand), i.e.* $\beta_3$ *increases, and the more the central bank is accommodative.*

A too aggressive interest rate change could be beneficial in the short run but wealth effects increase, and this policy could threaten the objective of mid-term stability. Furthermore, if the wealth effect become more important in the function of demand, as it is the case in many developed countries such as England or United States, a too aggressive monetary policy can be harmful to the whole economy because it degrade the wealth of economic agents, heavily indexed on interest rates with credit (variables-rate credits, subprime mortgages, variable-rate mortgage in United States particularly), which involves a sharp drop in demand since the weight of the effect wealth increases. This was shown by Bernanke and Gertler (1999) when they evoke the link between monetary policy and asset prices volatility. Finally, this phenomenon was/is the main issue of the American monetary policy (2007's summer). Many agents lose their wealth (property wealth for most), while the wealth effect is very present in the United States with borrowing capacity indexation on the valuation of their assets, which has destabilized the whole American economy, creating an uncertainty magnified to a drop in consumption, house prices, and financial markets in general.

To verify the consistency of our model, we continue this exercise in order to find simple mechanisms, by analyzing other parameters.

**Proposition 4 (4)** *The stronger is the past output in the future output, i.e.* $\beta_1$ *increases, and the more the central bank is restrictive, because* $\lambda_y$ *increases (and* $\lambda_\pi$ *does not change).*

Indeed, the more the output gap grows, and more the central bank "play its part" of stabilizer and increases its nominal rate. This was largely due to an increase of the optimal output gap coefficient.

**Proposition 5 (5)** *The stronger the demand (negative) depends to real interest rate, i.e.* $\beta_2$ *increases, and the more the central bank is accommodative.*

If the demand decreases, i.e. the elasticity of demand to real interest rate increases, then the central bank eases monetary conditions in order to revive growth.

**Proposition 6 (6)** *The stronger the elasticity of inflation to demand increases, i.e.* $\alpha_1$ *increases, and the more the central bank is restrictive.*

More inflation increases, and more the central bank tightens its optimal monetary policy. This classical result confirm the consistency of this basic one-country model. This result confirm that the central bank reacts to an increase in inflation, even if it's by the way of an increase in output impact, measured by $\alpha_1$.

**Proposition 7 (7)** *The stronger the policy discount factor increases, i.e. $\beta$ increases, and the more the central bank is restrictive.*

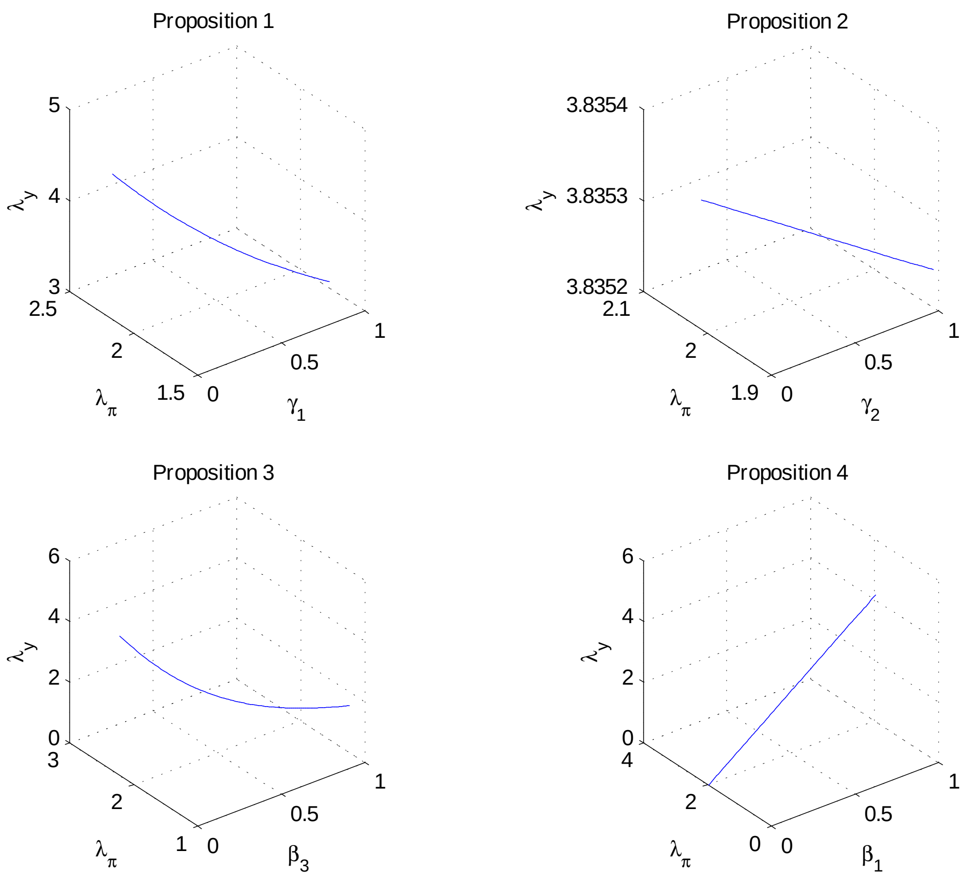
The the society's discount factor $\beta$ has generally a constant value that guarantees the safe numerical convergence of our model solutions. If we modify this constant parameter in the model, it involves an increasing in central bank interest rate because, if $\beta$ increases, the central bank loss function increases. In order to minimize this loss function, the central bank must augment in its interest rate instrument.

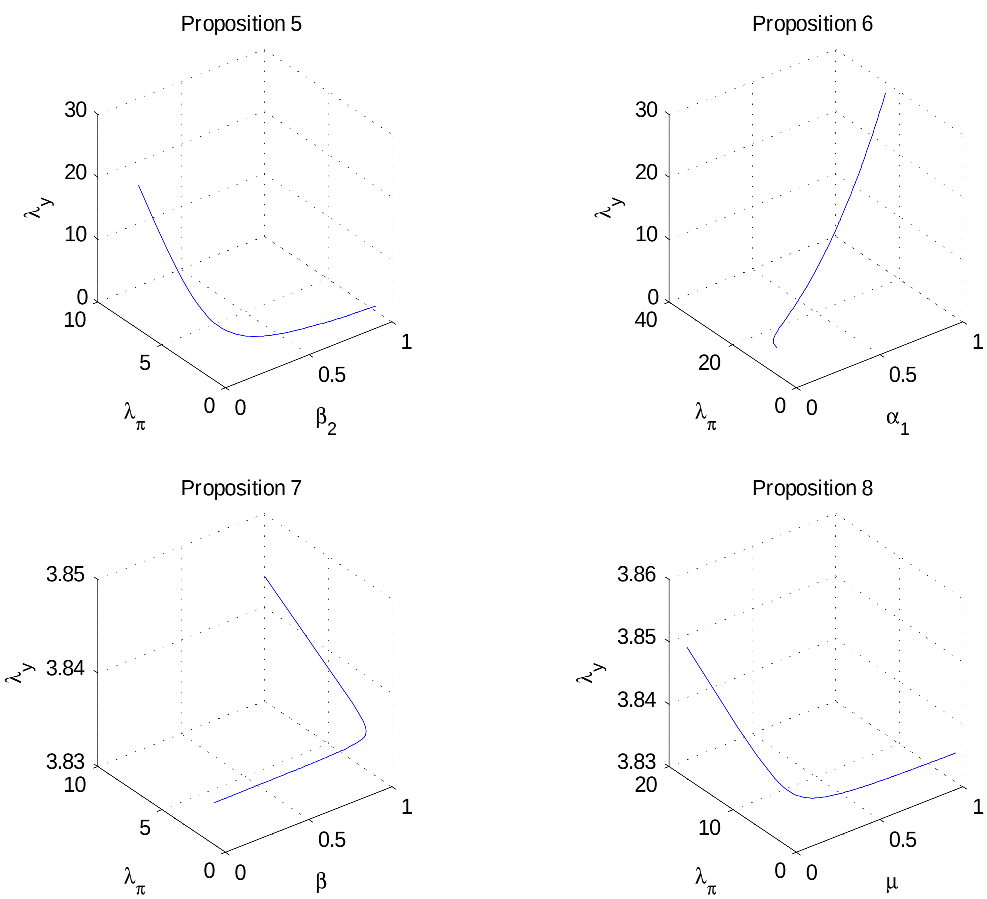
**Proposition 8 (8)** *The stronger the relative central bank's weight on output stabilization increases, i.e. $\mu$ increases, and the more the central bank is accommodative.*

It is well known the central bank interest rate instrument increase demand if it decrease. This result is thus a consequence of the negative relation between the demand and the interest rate.

We calibrate the model with parameters given by Walsh (1999) and Sodërlind (2007). The main idea of these graphs are to confirm propositions by a simple modelization. All propositions are verified.

This simple case one-country model don't take into account heterogeneities. But we use a similar methodology to solve the two-country model.


Proposition 1
Proposition 2
Proposition 3
Proposition 4
λy
λπ
γ1
γ2
β3
β1
3.8354
3.8353
3.8352


Proposition 5
30
20
10
0
10
5
0
0
0.5
1
λ_y
λ_π
β_2
Proposition 6
30
20
10
0
40
20
0
0
0.5
1
λ_y
λ_π
α_1
Proposition 7
3.85
3.84
3.83
10
5
0
0
0.5
1
λ_y
λ_π
β
Proposition 8
3.86
3.85
3.84
3.83
20
10
0
0
0.5
1
λ_y
λ_π
μ

# 3 The two-country model

We take equations (1), (2) and (3) of the previous basic model. We adapt the Fuhrer and Moore (1995) model to an open economy and we enhance our basic one-country model by adding as exogenous variable the output of the other country in the demand function of the considered country. We are in the Euro Area case, and it implies that there is no exchange rate and there is the same interest rate instrument, $i_t$, between countries of the Euro Area. We obtain the following equations:

$$\pi_{t+1} = \pi_t + \alpha_1 y_t + \varepsilon_{t+1} \tag{8}$$

$$\pi^*_{t+1} = \pi^*_t + \alpha_2 y^*_t + \varepsilon^*_{t+1} \tag{9}$$

$$y_{t+1} = \beta_1 y_t - \beta_2 \left(i_t - E_t \left[\pi_{t+1}\right]\right) + \beta_3 w_t + \beta_4 y^*_t + \eta_{t+1} \tag{10}$$

$$y^*_{t+1} = \beta_5 y^*_t - \beta_6 \left(i_t - E_t \left[\pi^*_{t+1}\right]\right) + \beta_7 w^*_t + \beta_8 y_t + \eta^*_{t+1} \tag{11}$$

$$w_{t+1} = -\gamma_1 \left(i_{t+1} - E_{t+1} \left[\pi_{t+2}\right]\right) + \gamma_2 E_{t+1} \left[y_{t+2}\right] + u_{t+1} \tag{12}$$

$$w^*_{t+1} = -\gamma_3 \left(i_{t+1} - E_{t+1} \left[\pi^*_{t+2}\right]\right) + \gamma_4 E_{t+1} \left[y^*_{t+2}\right] + u^*_{t+1} \tag{13}$$

where $\varepsilon_{t+1}$, $\varepsilon^*_{t+1}$, $\eta_{t+1}$, $\eta^*_{t+1}$, $u_{t+1}$ and $u^*_{t+1}$ are asymmetric exogenous random shocks, mutually uncorrelated *i.i.d.* processes with zero means and constant variances. We will use the system of equations (8) to (13) as our new model adapted to heterogeneous countries in Euro Area. Equations (8) and (9) are classical accelerationist Phillips curves. There is no interaction between the two countries in our model for inflation. This assumption could be further pursued. Based on Walsh (1997), Svensson (1997), and Fuhrer and Moore (1995) model, we enhance the demand function of an additional macroeconomic variable and by the demand of the foreign country (which is positively linked (empirically) to the domestic demand in the Euro Area framework). From a symmetrical view, we deduce the equations (10) and (11) of the model.

Continuing with the idea of Svensson (1997), we can consider this enhanced demand function as an exogenous variable. This exogenous variable is endogenized in order to determine the optimal reaction rule (by solving the central bank intertemporal optimization problem). Still in the path of Svensson (1997 and 2002), he also enhances the accelerationist Phillips curve by the same exogenous variable. We don't do so because the accelerationist Phillips curve is already impacted by wealth effects (one time), and it could be spurious to do more.

Finally, equations (12) and (13) are asset prices (log) for the two countries of our enhanced model. Interactions between foreign asset prices and domestic asset prices are not included in this model. However, this extension is plausible because of the opening of financial markets in the Euro Area. We follow the same methodology as to solve the basic model: we try to express the output gap and inflation gap expectations in terms of used variables. We have

$$E_t\left[y_{t+1}\right]=\delta_1 y_t+\delta_2\left(i_t-\pi_t\right)+\delta_3 y_t^* \tag{14}$$

and symetrically

$$E_t\left[y_{t+1}^*\right]=\delta_4 y_t^*+\delta_5\left(i_t-\pi_t^*\right)+\delta_6 y_t \tag{15}$$

By replacing in (10) the expression of expected inflation, given by (8), and the expression of the wealth effects, given by (12), we have:

$$\begin{aligned} y_{t+1} &= \left(\beta_1+\alpha_1\left(\beta_2+\beta_3\gamma_1\right)\right) y_t-\left(\beta_2+\beta_3\gamma_1\right)\left(i_t-\pi_t\right) \\ &\quad +\beta_3\gamma_2 E_t\left[y_{t+1}\right]+\beta_4 y_t^*+\eta_{t+1} \end{aligned} \tag{16}$$

So based on (14), we have:

$$\begin{aligned} y_{t+1} &= \left(\beta_1+\alpha_1\beta_2+\beta_3\left(\alpha_1\gamma_1+\gamma_2\delta_1\right)\right) y_t-\left(\beta_2+\beta_3\left(\gamma_1+\gamma_2\delta_2\right)\right)\left(i_t-\pi_t\right) \\ &\quad +\left(\beta_4+\beta_3\gamma_2\delta_3\right) y_t^*+\eta_{t+1} \end{aligned} \tag{17}$$

In a business area, like Euro Area, trades are principally made with other partners from the same area. We can assume that macroeconomic factors follows other countries macroeconomic factors. The link between one country and the other is called here heterogeneity. In fact, there is not only trades between countries to say that there is a link. Two countries are strongly different even if they have same trades (different fiscal policy, different financial systems). We could model heterogeneities by the following system:

$$\left\{\begin{array}{l} y_t^*=\sigma_y y_t+\xi_t \\ r_t^*=i_t-\pi_t^*=\sigma_\pi\left(i_t-\pi_t\right)+\psi_t \end{array}\right. \tag{18}$$

where $\sigma_y$ and $\sigma_\pi$ are coefficients which modeling heterogeneities. If $\sigma_y=\sigma_\pi=1$, then there is no heterogeneity on average. $\xi_t$ and $\psi_t$ represent exogenous random shocks respectively to output (gap) and real interest rate. We assume that they are mutually uncorrelated *i.i.d.* processes with zero means and constant variances. Finally, we have a demand equation of the following form[8]:

$$y_{t+1}=\left(\frac{A+F+\sigma_y\left(C+D\right)}{1+\sigma_y}\right) y_t-\left(\frac{B+\sigma_\pi E}{1+\sigma_y}\right)\left(i_t-\pi_t\right)+\varpi_{t+1} \tag{19}$$

The problem is summarized by the following system (as in the basic model) :

$$\left\{\begin{array}{l} y_{t+1}=\varphi_t^{\prime}+\varpi_{t+1} \\ \pi_{t+1}=\kappa_t+\varepsilon_{t+1} \end{array}\right. \tag{20}$$

with $\varphi_t^{\prime}=\left(\frac{A+F+\sigma_y(C+D)}{1+\sigma_y}\right) y_t-\left(\frac{B+\sigma_\pi E}{1+\sigma_y}\right)\left(i_t-\pi_t\right)$ and $\kappa_t=\pi_t+\alpha_1 y_t$.

[8] See Appendix 8 for more details, and a definition of $A$, $B$, $C$, $D$ and $E$.

We assume, may be strongly, that $\varpi_{t+1}$ represent an exogenous random shock to enhanced output, since it is a linear combination of exogenous random shocks, mutually uncorrelated *i.i.d.* processes with zero means and constant variances. The resolution of the enhanced model (two-countries) is then similar with the resolution of the basic model (one country) in terms of methodology and thus: $c\kappa_t = \varphi_t^{'}$. We have: $i_t = \lambda_\pi^{'}\pi_t + \lambda_y^{'}y_t$ what is the general form of the Taylor rule. We verify our model without heterogeneities, i.e. with $\sigma_y = \sigma_\pi = 1$, and we get the same results than the basic one-country model[9].

## 3.1 Interpretation of theoretical results

As in the basic model, we compute partial derivatives of Taylor rule coefficients with respect to used parameters. Results are too complex to determine partial derivatives signs. We must proceed a numerical analysis in order to obtain consistent conclusions.

This two-country model can be extended to $n$ countries. Computation would be tedious and would lead to more uncertainties in the analysis of the effects of parameters on optimal monetary policy coefficients. The Appendix 11 presents the results of classical cases, with and without wealth effects for both one and two-country model (Mathematica). These calculations are needed to verify models in order to justify their mathematical consistency.

## 3.2 Interpretation of empirical results

We choose two country to estimate the monetary policy rule: France and Germany. We assume that all the right-hand side regressors of equations (8) to (13) are exogenous. The errors are heteroskedastic and contemporaneously correlated. The seemingly unrelated regression (SUR) method, also known as the multivariate regression, or Zellner's method, estimates the parameters of our system of equations (8) to (11), accounting for heteroskedasticity and contemporaneous correlation in the errors across equations. The estimates of the cross-equation covariance matrix are based upon parameter estimates of the unweighted system.

We compute the estimation with 107 monthly year-over-year growth rates observations (from January 1999 to December 2007).

[9] See Appendix 10 for more details.

| | Coefficient | Standard Error | Student Test | p-value |
|---|---|---|---|---|
| $\alpha_1$ | 0.00311 | 0.00029 | 10.69 | 0.00001 |
| $\beta_1$ | 0.95240 | 0.00109 | 874.74 | 0 |
| $\beta_2$ | 0.01410 | 0.00105 | 13.46 | 0.00001 |
| $\beta_3$ | 0.00335 | 0.00016 | 21.38 | 0.00001 |
| $\beta_4$ | 0.02086 | 0.00129 | 16.21 | 0.00001 |
| $\alpha_2$ | 0.00773 | 0.00016 | 52.53 | 0.00001 |
| $\beta_5$ | 0.89602 | 0.00302 | 297.02 | 0 |
| $\beta_6$ | 0.10332 | 0.00197 | 52.36 | 0.00001 |
| $\beta_7$ | 0.00662 | 0.00017 | 39.95 | 0.00001 |
| $\beta_8$ | $-0.01320$ | 0.00030 | $-44.19$ | 0.00001 |

Determinant residual covariance: 7.179e-21

Similarly, we estimate heterogeneity coefficients, $\sigma_y$ and $\sigma_\pi$, and wealth effect equations, (12) and (13), in order to calibrate the model. We obtain the following results:

$$\begin{aligned}
\sigma_y &= 0.659894 \\
\sigma_\pi &= 0.623910 \\
\gamma_1 &= 2.826870 \\
\gamma_2 &= 2.586976 \\
\gamma_3 &= 4.991488 \\
\gamma_4 &= 4.166464
\end{aligned}$$

With all these parameters, we obtain $\lambda_\pi^{'} = 0.8873$ and $\lambda_y^{'} = 0.4438$. Without heterogeneities, we find that Taylor rule coefficients are stronger than the case with heterogeneities. More qualitatively, we obtain $\lambda_\pi^{'} = 0.9312$ and $\lambda_y^{'} = 0.4697$ in the case without heterogeneities. We conclude that more heterogeneities are far from one and more Taylor rule's coefficients are lower.

**Proposition 9** *Increasing output heterogeneity ($\sigma_y$) involves a restrictive monetary policy and increasing inflation heterogeneity ($\sigma_y$) involves an accommodating monetary policy.*

We verify the impact of wealth effect of one of the two country on Taylor rule coefficients. It confirm the **Proposition 3** of the one-coutry model. But if the wealth effect coefficients ($\beta_3$ and $\beta_7$) evolves in the same direction, the accommodative behavior of the central bank is less pronounced. It confirm the heterogeneity propositions.

## 3.3 Critics and reformulation

One can do it better: the formulation of countries heterogeneities with heterogeneities' coefficients makes the system undetermined with more equation than unknown variables (more precisely, it makes the system of equation over identified). More of that, in the two-country model, we focus on parameters and we analyze the model when these parameters moves. Following DSGE modelling, we focus now on stochastic processes and shocks which moves variables. We consider the following model:

$$\pi_{t+1} = \pi_t + \alpha_1 y_t + \varepsilon_{t+1}$$

$$\pi^*_{t+1} = \pi^*_t + \alpha_2 y^*_t + \varepsilon^*_{t+1}$$

$$y_{t+1} = \beta_1 y_t - \beta_2 \left(i_t - E_t \left[\pi_{t+1}\right]\right) + \beta_3 w_t + \beta_4 y^*_t + \eta_{t+1}$$

$$y^*_{t+1} = \beta_5 y^*_t - \beta_6 \left(i_t - E_t \left[\pi^*_{t+1}\right]\right) + \beta_7 w^*_t + \beta_8 y_t + \eta^*_{t+1}$$

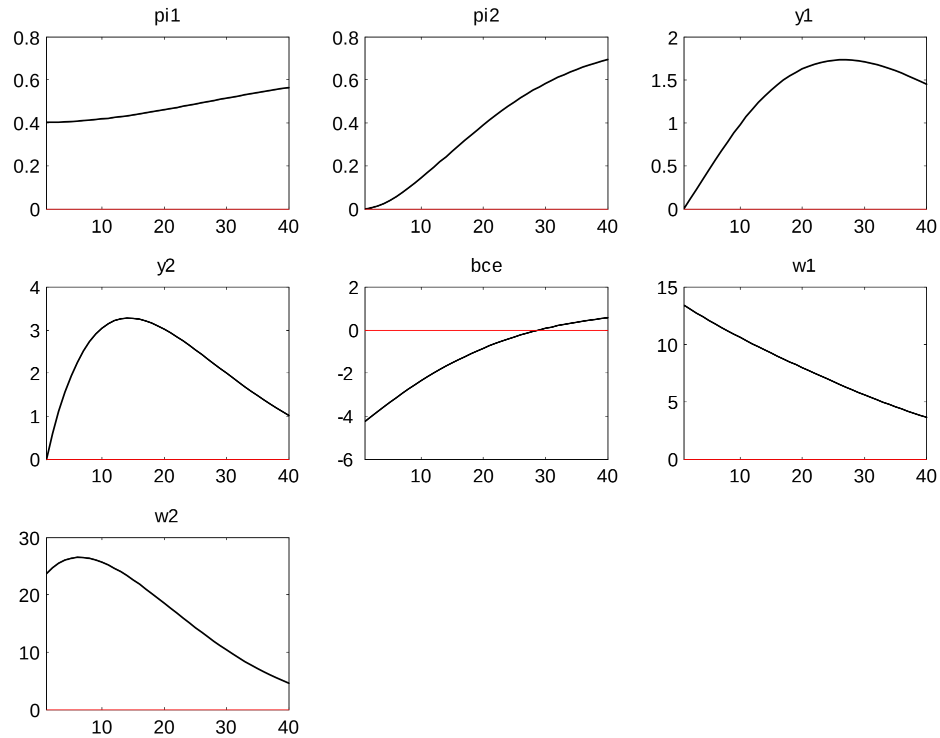
$$w_{t+1} = -\gamma_1 \left(i_{t+1} - E_{t+1} \left[\pi_{t+2}\right]\right) + \gamma_2 E_{t+1} \left[y_{t+2}\right] + u_{t+1}$$

$$w^*_{t+1} = -\gamma_3 \left(i_{t+1} - E_{t+1} \left[\pi^*_{t+2}\right]\right) + \gamma_4 E_{t+1} \left[y^*_{t+2}\right] + u^*_{t+1}$$

$$i_t = 0.05 \left(\lambda'_\pi \pi_t + \lambda'_y y_t\right) + 0.95 i_{t-1}$$

with the same variables and parameters as previously. For convenience, we follow Woodford (1999) interest rate smoothing with a parameter of 0.95 as in the literature about euro area monetary policy.

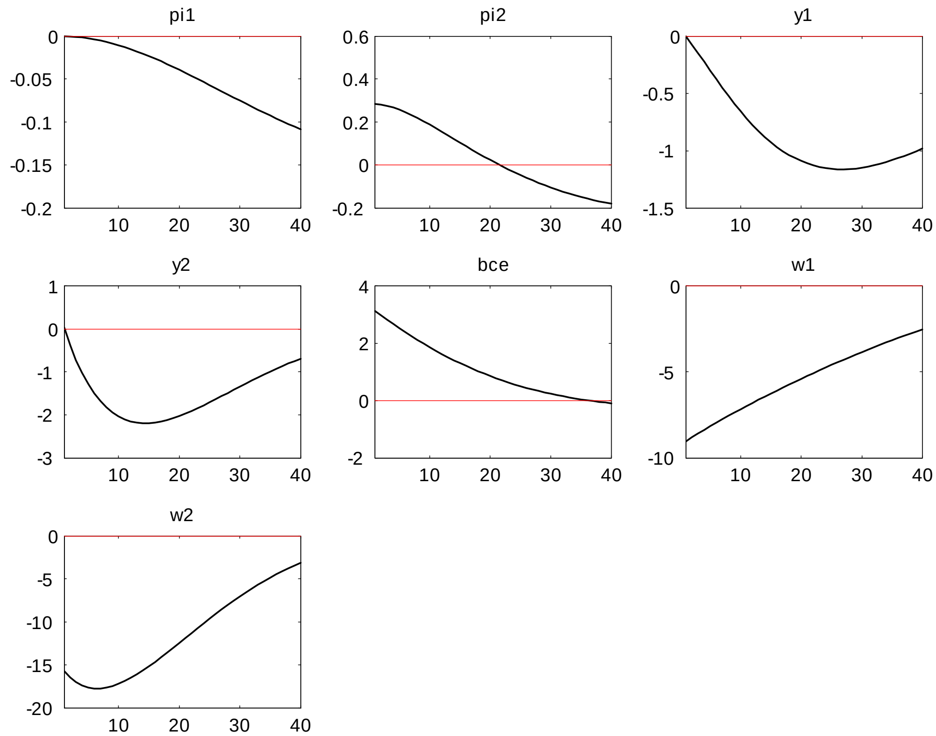
We compute a DSGE (Dynamic Stochastic General Equilibrium) estimation and we obtain IRF (Impulse Response Functions). Since the seminal work by Rotemberg and Woodford (1997), there has been an increasing interest in estimating macroeconomic models by using Impulse Response Function (IRF) matching estimators. The method is appealing because of its simplicity, and because it focuses on estimating the parameters on the basis of impulse responses, which directly capture the dynamics that are of primary interest to macroeconomists. Among the recent papers that have used IRF matching estimators we have: Christiano Eichenbaum and Evans (2005), Altig et al. (2004), Jordà and Kozicki (2005), Boivin and Giannoni (2006), Uribe and Yue (2006), DiCecio (2005) and DiCecio and Nelson (2006).




Shock to $\pi_{t+1}$

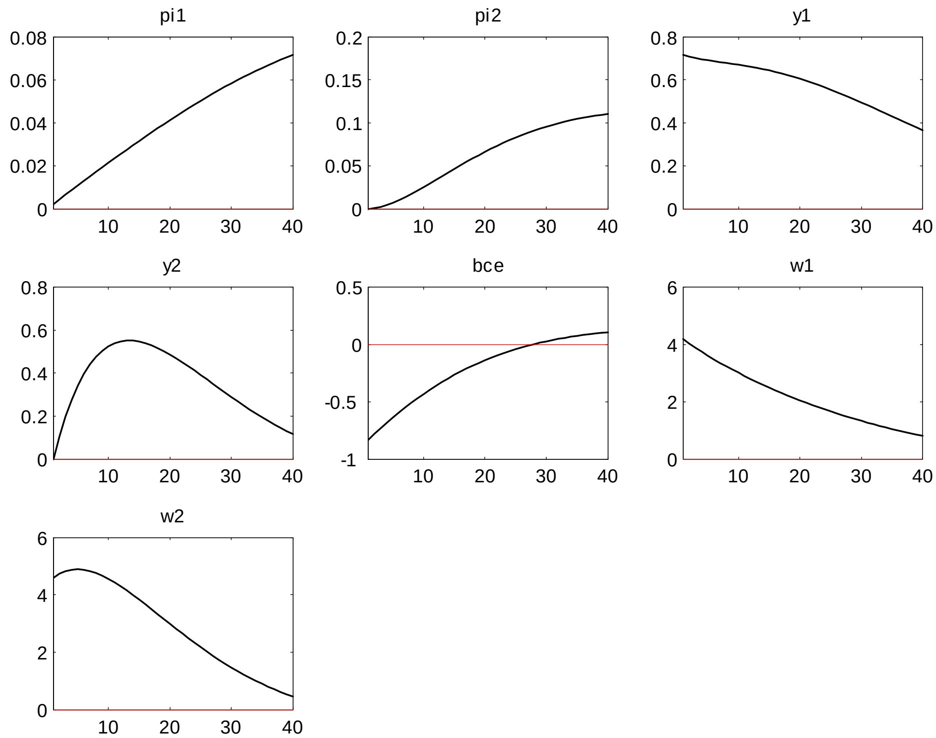
When a shock to inflation happen in the home country (France in our empirical work), inflation in the other country (Germany) grow up. Output in Germany increase twice more than in France and the ECB increase its interest rate in order to stop inflation. Wealth effects decrease in the two countries.

the other country



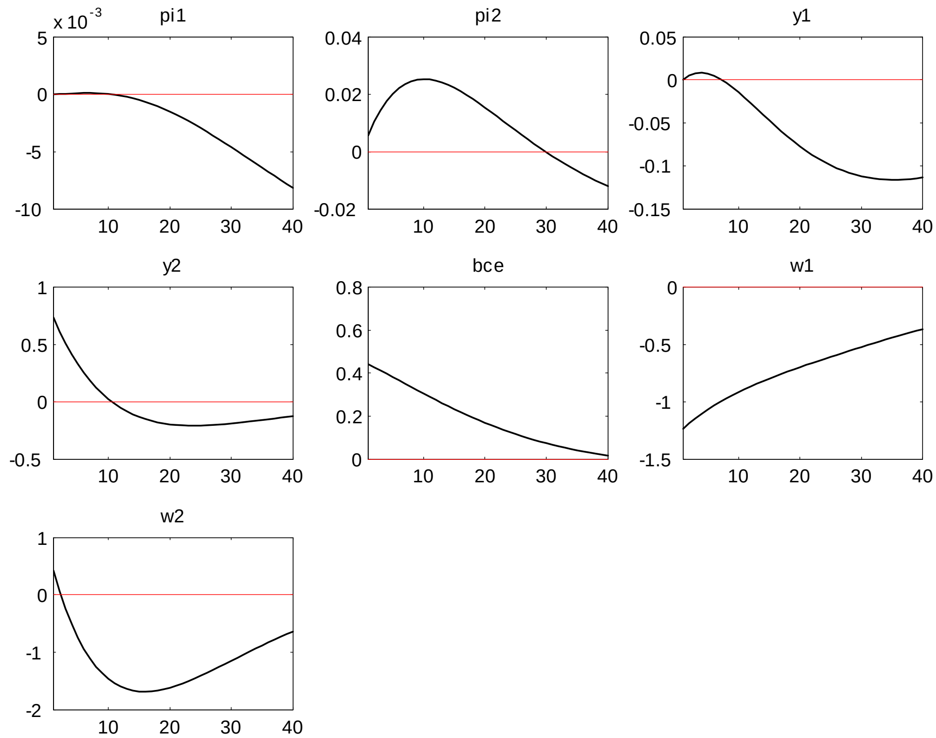

Shock to $\pi^*_{t+1}$

A similar inflation shock on Germany involve a decrease in output and in ECB interest rates. This drive up wealth effects in all countries.



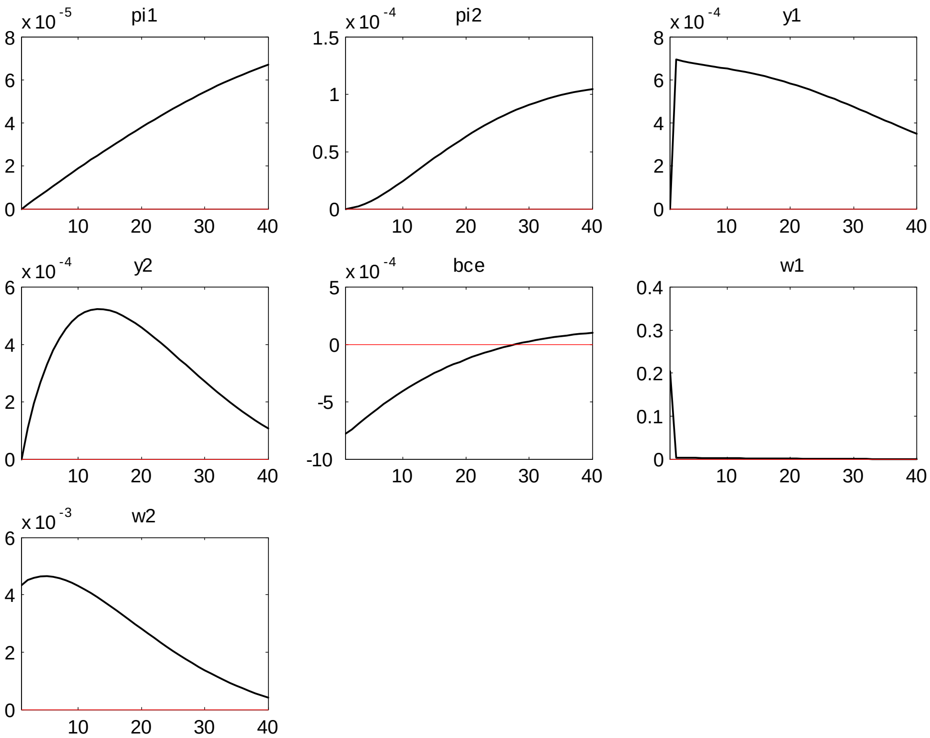

Shock to $y_{t+1}$

A shock in French output increase German output and involve an stronger inflation in all countries. ECB interest rates then increase and wealth effects are diminishing there value.



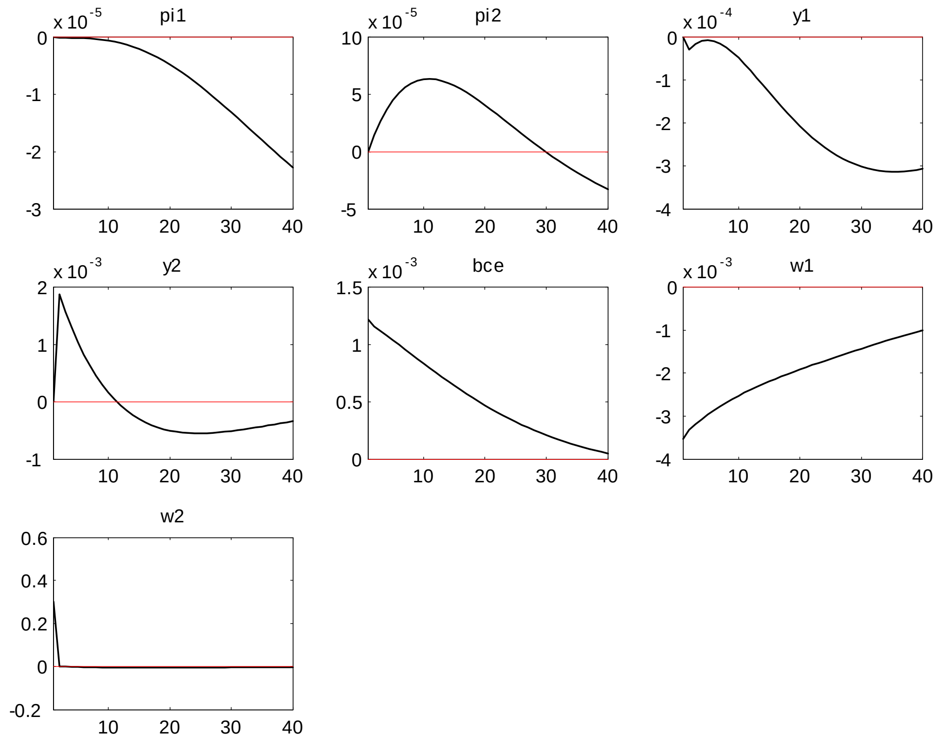

Shock to $y^*_{t+1}$

A similar output shock in Germany decrease French output and inflation in all countries. ECB interest rates increasing and wealth effects starts there increasing movement.




Shock to $W_{t+1}$

A shock on French wealth effects increase output and inflation in all countries then inducing a restrictive monetary policy from the ECB.




Shock to $W^*_{t+1}$

A shock to German wealth effect involve an increasing French wealth effect. But output and inflation of all countries move down and ECB conduct an accommodative monetary policy to stabilize the Euro area economy. The bad news is the French wealth effect which increase slowly to its equilibrium level. It may be dangerous in the long run.

# 4 Optimal monetary policy

The two models above have several problems. First, their equations are not those of the classical New Keynesian framework. Although Furher (1997) has proved that forward looking is not significantly different with American data than backward looking for inflation and output gap equations, literature tend to use forward looking variables.

# 5 Conclusion

Although there is global approval as to wether the central bank reacts to asset prices inflation, there is still a broad consensus on the important role played by asset prices in the transmission of the monetary policy. This paper is based on the assumption that earnings from asset prices (dividends and valuation) have an impact on aggregate demand and (thus) on inflation. This macroeconomic structural model, which comes mainly from Svensson (1997, 2002), Walsh (1998) and Bernanke and Gertler (1999), allows the central bank to solve the intertemporal optimization problem of minimizing the weighted sum of the output gap variance and the inflation targeting (gap) variance. Compared to a classical Taylor rule, we find that the presence of wealth effects implies a more accommodative central bank behavior, in order not to destabilize the (mid-term) economic dynamics. Similarly, the higher is the sensitivity of financial markets with respect to real interest rates, and the more the monetary authority should pursue an accommodative policy, in order to agents' wealth (and the demand in our model) is not affected.

The two-country model show that more heterogeneities coefficients of output between France and Germany are stronger, more the central bank should be restrictive in order to pursue an optimal monetary policy. But more heterogeneities coefficients of inflation between France and Germany are stronger, more the central bank should be accommodative.

It would be interesting to consider an affine heterogeneity function, differently from what is presented here (different from the linear function which facilitates calculations).

# 6 Appendixes

## 6.1 Appendix 1: Monetary policy as optimal control problem

By replacing equation (3) in the equation (2) we obtain:

$$y_{t+1} = \beta_1 y_t + \beta_3 \gamma_2 E_t\left[y_{t+1}\right] - \left(\beta_2 + \beta_3 \gamma_1\right)\left(i_t - E_t\left[\pi_{t+1}\right]\right) + \upsilon_{t+1} \tag{21}$$

where $\upsilon_{t+1} = \beta_3 u_t + \eta_{t+1}$ is the composite output shock : we remark that this composite demand shock is composed by an asset prices shock, $u_t$, and by a purely demand shock, $\eta_{t+1}$. The magnitude of asset prices impact on production depends on the wealth effects size, $\beta_3$. We take expectation conditionally to $t$ of the expression (1) and (21):

$$E_t\left[\pi_{t+1}\right] = \pi_t + \alpha_1 y_t \tag{22}$$

$$E_t\left[y_{t+1}\right] = \beta_1 y_t + \beta_3 \gamma_2 E_t\left[y_{t+1}\right] - \left(\beta_2 + \beta_3 \gamma_1\right)\left(i_t - E_t\left[\pi_{t+1}\right]\right) \tag{23}$$

We put equation (22) in the equation (23) and we have:

$$E_t\left[y_{t+1}\right] = \frac{\beta_1 + \alpha_1\left(\beta_2 + \beta_3 \gamma_1\right)}{1 - \beta_3 \gamma_2} y_t - \frac{\beta_2 + \beta_3 \gamma_1}{1 - \beta_3 \gamma_2}\left(i_t - \pi_t\right) \tag{24}$$

and we have the expressions of $\delta_1$ and $\delta_2$:

$$\delta_1 = \frac{\beta_1 + \alpha_1 (\beta_2 + \beta_3 \gamma_1)}{1 - \beta_3 \gamma_2}$$

$$\delta_2 = \frac{\beta_2 + \beta_3 \gamma_1}{1 - \beta_3 \gamma_2}$$

By replacing the expression of $E_t [y_{t+1}]$ given by the equation (24) in equation (21), we obtain:

$$y_{t+1} = (\beta_1 + \alpha_1 (\beta_2 + \beta_3 \gamma_1) + \beta_3 \gamma_2 \delta_1) y_t - (\beta_2 + \beta_3 \gamma_1 + \beta_3 \gamma_2 \delta_2) (i_t - \pi_t) + \upsilon_{t+1} \tag{25}$$

and we have the expressions of $A$ and $B$:

$$A = \beta_1 + \alpha_1 (\beta_2 + \beta_3 \gamma_1) + \beta_3 \gamma_2 \delta_1$$

$$B = \beta_2 + \beta_3 \gamma_1 + \beta_3 \gamma_2 \delta_2$$

We obtain an equation of the following form:

$$y_{t+1} = \varphi_t + \nu_{t+1}$$

where $\varphi_t = A y_t - B (i_t - \pi_t)$. Following Walsh (1998), $\pi_t$ and $y_t$ are predetermined when $i_t$ is fixed. We can define $\varphi_t$ as the central bank's control variable and $\kappa_t$ as the state variable of the system, and we obtain the following system:

$$\begin{aligned} \pi_{t+1} &= \kappa_t + \varepsilon_{t+1} \\ y_{t+1} &= \varphi_t + \nu_{t+1} \end{aligned}$$

### 6.2 Appendix 2: Solving the optimization problem

We solve the following minimization problem:

$$V(\kappa_t) = \min_{\varphi_t} E_t \left\{ \frac{1}{2} \left[ (\kappa_t + \varepsilon_{t+1})^2 + \mu (\varphi_t + \nu_{t+1})^2 \right] + \beta V(\kappa_t + \alpha_1 \varphi_t + \omega_{t+1}) \right\}$$

The first order condition is: $\frac{\partial V(\kappa_t)}{\partial \varphi_t} = 0$ and thus:

$$\mu \varphi_t + \alpha_1 \beta E_t \left[ \frac{\partial V(\kappa_t + \alpha_1 \varphi_t + \omega_{t+1})}{\partial \varphi_t} \right] = 0 \tag{26}$$

We use the envelope theorem to have an expression of
$E_t \left[ \frac{\partial V(\kappa_t + \alpha_1 \varphi_t + \omega_{t+1})}{\partial \varphi_t} \right] = E_t \left[ V_{\varphi_t} (\kappa_t + \alpha_1 \varphi_t + \omega_{t+1}) \right]$:
$\frac{\partial V(\kappa_t)}{\partial \kappa_t} = E_t \left[ \frac{\partial}{\partial \kappa_t} \left( \frac{1}{2} \left[ (\kappa_t + \varepsilon_{t+1})^2 + \mu (\varphi_t + \nu_{t+1})^2 \right] + \beta V(\kappa_t + \alpha_1 \varphi_t + \omega_{t+1}) \right) \right]$
Thus: $V_{\kappa_t}(\kappa_t) d\kappa_t = E_t [\kappa_t + \beta V_{\kappa_t}(\kappa_t + \alpha_1 \varphi_t + \omega_{t+1})] d\kappa_t$
Thus: $V_{\kappa_t}(\kappa_t) = \kappa_t + \beta E_t [V_{\kappa_t}(\kappa_t + \alpha_1 \varphi_t + \omega_{t+1})]$
Thus: $\alpha_1 \beta E_t [V_{\kappa_t}(\kappa_t + \alpha_1 \varphi_t + \omega_{t+1})] = \alpha_1 (V_{\kappa_t}(\kappa_t) - \kappa_t)$

By replacing in equation (26), we obtain: $\mu\varphi_t + \alpha_1 \left(V_{\kappa_t}\left(\kappa_t\right) - \kappa_t\right) = 0$
We deduce that: $E_t\left[V_{\kappa_t}\left(\kappa_t + \alpha_1\varphi_t + \omega_{t+1}\right)\right] = E_t\left[\kappa_t + \alpha_1\varphi_t + \omega_{t+1}\right] - \frac{\mu}{\alpha_1} E_t\left[\varphi_{t+1}\right]$
But: $E_t\left[\kappa_{t+1}\right] = E_t\left[\kappa_t + \alpha_1\varphi_t + \omega_{t+1}\right] = \kappa_t + \alpha_1\varphi_t$
Then: $E_t\left[V_{\kappa_t}\left(\kappa_t + \alpha_1\varphi_t + \omega_{t+1}\right)\right] = \kappa_t + \alpha_1\varphi_t - \frac{\mu}{\alpha_1} E_t\left[\varphi_{t+1}\right]$
By replacing this expression in equation (26), we have:

$$\mu\varphi_t + \alpha_1\beta\left(\kappa_t + \alpha_1\varphi_t - \frac{\mu}{\alpha_1} E_t\left[\varphi_{t+1}\right]\right) = 0$$

Thus:

$$\varphi_t = \frac{\beta\mu}{\mu + \beta\alpha_1^2} E_t\left[\varphi_{t+1}\right] - \frac{\alpha_1\beta}{\mu + \beta\alpha_1^2}\kappa_t \tag{27}$$

When the monetary policy is fixed at time $t$, $\kappa_t$ summarize the state of the system, i.e. the optimal policy, giving a linear quadratic structure of the following form: $\varphi_t = c\kappa_t$.

Thus, $\varphi_t = c\left(1 + \alpha_1 c\right)\kappa_t$ because:
$E_t\left[\varphi_{t+1}\right] = cE_t\left[\kappa_t + \alpha_1\varphi_t + \omega_{t+1}\right] = c\left(\kappa_t + \alpha_1\varphi_t\right) = c\left(\kappa_t + \alpha_1 c\kappa_t\right)$
By replacing in equation (27), we have:
$c\kappa_t = \frac{\beta\mu}{\mu+\beta\alpha_1^2} c\left(1 + \alpha_1 c\right)\kappa_t - \frac{\alpha_1\beta}{\mu+\beta\alpha_1^2}\kappa_t$
Thus: $\alpha_1\beta\mu c^2 + \left(\beta\mu - \mu - \beta\alpha_1^2\right)c - \alpha_1\beta = 0$

Then, $\Delta > 0$ and in order to fulfill the stability criterion of the process , we accept on accept only negative solution of this equation. Indeed, $\kappa_t + \alpha_1\varphi_t + \omega_{t+1} = \left(1 + \alpha_1 c\right)\kappa_t + \omega_{t+1}$. Thus the stability of the inflation process requires the following condition: $\left|1 + \alpha_1 c\right| < 1$ i.e. $c < 0$ because $\alpha_1 > 0$.

Thus, $c = \frac{\mu - \beta\mu - \beta\alpha_1^2 - \sqrt{\left(\beta\mu - \mu + \beta\alpha_1^2\right)^2 + 4\alpha_1^2\beta^2\mu}}{2\alpha_1\beta\mu} < 0$.

Finally, we obtain the following system: $\left\{\begin{array}{c} \varphi_t = Ay_t - B\left(i_t - \pi_t\right) \\ \varphi_t = c\kappa_t \end{array}\right.$

### 6.3 Appendix 3: Taylor rule

From Appendix 2, we obtain the following equation:

$$c\kappa_t = Ay_t - B\left(i_t - \pi_t\right)$$

and we know that:

$$\kappa_t = \pi_t + \alpha_1 y_t$$

$$A = \beta_1 + \alpha_1\left(\beta_2 + \beta_3\gamma_1\right) + \beta_3\gamma_2\delta_1$$

$$B = \beta_2 + \beta_3\gamma_1 + \beta_3\gamma_2\delta_2$$

$$\delta_1 = \frac{\beta_1 + \alpha_1\left(\beta_2 + \beta_3\gamma_1\right)}{1 - \beta_3\gamma_2}$$

$$\delta_2 = \frac{\beta_2 + \beta_3\gamma_1}{1 - \beta_3\gamma_2}$$

Then, we have the following Taylor rule:

$$i_t = \lambda_\pi \pi_t + \lambda_y y_t$$

where $\lambda_\pi = \frac{\beta_2 - c + \beta_3(\gamma_1 + \gamma_2 c)}{\beta_2 + \beta_3\gamma_1}$ and $\lambda_y = \frac{\gamma_1(\beta_2 - c) + \beta_1 + \alpha_1\beta_3(\gamma_1 + \gamma_2 c)}{\beta_2 + \beta_3\gamma_1}$

This equation is similar to a Taylor rule, where the nominal interest rate is a linear function of the output gap and inflation targeting.

## 6.4 Appendix 4: Standard / Wealth Effect Taylor rule coefficient of inflation

It is obvious that $\frac{1-a}{1+b} < 1$ with $\{a; b\} \in ]0; +\infty[$

Taking $a = \beta_3\gamma_2$ and $b = \frac{\beta_3\gamma_1}{\beta_2}$, thus $\frac{1-\beta_3\gamma_2}{1+\frac{\beta_3\gamma_1}{\beta_2}} < 1$.

Thus $\frac{1-\beta_3\gamma_2}{\beta_2+\beta_3\gamma_1} < \frac{1}{\beta_2}$ because $\beta_2 > 0$.

Thus $\frac{-c(1-\beta_3\gamma_2)}{\beta_2+\beta_3\gamma_1} < -\frac{c}{\beta_2}$ because $c < 0$ and thus $1 - \frac{c(1-\beta_3\gamma_2)}{\beta_2+\beta_3\gamma_1} < 1 - \frac{c}{\beta_2}$ so $\lambda_\pi < \lambda_\pi^s$.

## 6.5 Appendix 5: Standard / Wealth Effect Taylor rule coefficient of production

• If $\beta_1 \neq 0$

We have $\frac{1}{c} < \frac{\alpha_1(\gamma_1+\beta_2\gamma_2)}{\beta_1\gamma_1}$ because $c < 0$ and $\frac{\alpha_1(\gamma_1+\beta_2\gamma_2)}{\beta_1\gamma_1} > 0$.
Thus $\beta_1\gamma_1 > \alpha_1\beta_2\gamma_2 c + \alpha_1\gamma_1 c$ because $c < 0$.
Thus $\beta_1\gamma_1 - \alpha_1\gamma_1 c > \alpha_1\beta_2\gamma_2 c$.
Thus $\beta_1\beta_3\gamma_1 - \alpha_1\beta_3\gamma_1 c > \alpha_1\beta_2\beta_3\gamma_2 c$ because $\beta_3 > 0$.
Thus $\beta_1\beta_2 + \beta_1\beta_3\gamma_1 - \alpha_1\beta_2 c - \alpha_1\beta_3\gamma_1 c > \beta_1\beta_2 + \alpha_1\beta_2\beta_3\gamma_2 c - \alpha_1\beta_2 c$.
Thus $(\beta_1 - \alpha_1 c)(\beta_2 + \beta_3\gamma_1) > \beta_2(\beta_1 - \alpha_1 c(1 - \beta_3\gamma_2))$
Thus $\alpha_1 + \frac{\beta_1 - \alpha_1 c}{\beta_2} > \alpha_1 + \frac{\beta_1 - \alpha_1 c(1-\beta_3\gamma_2)}{\beta_2+\beta_3\gamma_1}$ because $\beta_2 + \beta_3\gamma_1 > 0$ and $\beta_2 > 0$.
Thus $\lambda_y^s > \lambda_y$ if $\beta_1 \neq 0$.

• If $\beta_1 = 0$

We have $\lambda_y^s = \alpha_1 - \frac{c\alpha_1}{\beta_2}$ and $\lambda_y = \alpha_1 + \frac{\beta_1 - \alpha_1 c(1-\beta_3\gamma_2)}{\beta_2+\beta_3\gamma_1}$
But $\gamma_1 > -\beta_2\gamma_2$ because $\gamma_1 > 0$ and $\beta_2\gamma_2 > 0$, thus $\beta_2 + \beta_3\gamma_1 > 0$
Thus $\frac{1}{\beta_2} > \frac{1-\beta_3\gamma_2}{\beta_2+\beta_3\gamma_1}$ because $\beta_2 > 0$ and $\beta_2 + \beta_3\gamma_1 > 0$
Thus $\alpha_1 - \frac{c\alpha_1}{\beta_2} > \alpha_1$ because $c < 0$ and thus $\lambda_y^s > \lambda_y$ if $\beta_1 \neq 0$.

## 6.6 Appendix 6: Dynamics of Taylor rule coefficients

We assume that $c < 0$, $\beta_3\,\gamma_2 < 1$, $\frac{\partial c}{\partial \alpha_1} < 0$, $\frac{\partial c}{\partial \beta} < 0$ and $\frac{\partial c}{\partial \mu} > 0$.

With $\lambda_\pi = \frac{\beta_2 - c + \beta_3(\gamma_1 + \gamma_2 c)}{\beta_2 + \beta_3\gamma_1}$ and $\lambda_y = \frac{\gamma_1(\beta_2 - c) + \beta_1 + \alpha_1\beta_3(\gamma_1 + \gamma_2 c)}{\beta_2 + \beta_3\gamma_1}$, we obtain the following results:

$$
\begin{array}{ll}
\frac{\partial\lambda_y}{\partial\beta_1} = \frac{1}{\beta_2+\beta_3\,\gamma_1} > 0 & \frac{\partial\lambda_\pi}{\partial\beta_1} = 0 \\
\frac{\partial\lambda_y}{\partial\beta_2} = \frac{-\beta_1+\alpha_1\,c(1-\,\beta_3\,\gamma_2)}{(\beta_2+\beta_3\,\gamma_1)^2} < 0 & \frac{\partial\lambda_\pi}{\partial\beta_2} = \frac{c(1-\,\beta_3\,\gamma_2)}{(\beta_2+\beta_3\,\gamma_1)^2} < 0 \\
\frac{\partial\lambda_y}{\partial\beta_3} = \frac{-(\beta_1\,\gamma_1)+c\,\alpha_1\,(\gamma_1+\beta_2\,\gamma_2)}{(\beta_2+\beta_3\,\gamma_1)^2} < 0 & \frac{\partial\lambda_\pi}{\partial\beta_3} = \frac{c\,(\gamma_1+\beta_2\,\gamma_2)}{(\beta_2+\beta_3\,\gamma_1)^2} < 0 \\
\frac{\partial\lambda_y}{\partial\gamma_1} = -\frac{\beta_3\,(\beta_1-c\,\alpha_1\,(1-\beta_3\,\gamma_2))}{(\beta_2+\beta_3\,\gamma_1)^2} < 0 & \frac{\partial\lambda_\pi}{\partial\gamma_1} = \frac{c\,\beta_3\,(1-\beta_3\,\gamma_2)}{(\beta_2+\beta_3\,\gamma_1)^2} < 0 \\
\frac{\partial\lambda_y}{\partial\gamma_2} = \frac{c\,\alpha_1\,\beta_3}{\beta_2+\beta_3\,\gamma_1} < 0 & \frac{\partial\lambda_\pi}{\partial\gamma_1} = \frac{c\,\beta_3}{\beta_2+\beta_3\,\gamma_1} < 0 \\
\frac{\partial\lambda_y}{\partial\alpha_1} = -\frac{1-\beta_3\,\gamma_2}{\beta_2+\beta_3\,\gamma_1}\left(\frac{\partial c}{\partial\alpha_1}\right) > 0 & \frac{\partial\lambda_\pi}{\partial\alpha_1} = 1-\frac{1-\beta_3\,\gamma_2}{\beta_2+\beta_3\,\gamma_1}\left(\frac{\partial c}{\partial\alpha_1}\right) > 0 \\
\frac{\partial\lambda_y}{\partial\mu} = -\frac{1-\beta_3\,\gamma_2}{\beta_2+\beta_3\,\gamma_1}\left(\frac{\partial c}{\partial\mu}\right) < 0 & \frac{\partial\lambda_\pi}{\partial\mu} = -\alpha_1\frac{1-\beta_3\,\gamma_2}{\beta_2+\beta_3\,\gamma_1}\left(\frac{\partial c}{\partial\mu}\right) < 0 \\
\frac{\partial\lambda_y}{\partial\beta} = -\frac{1-\beta_3\,\gamma_2}{\beta_2+\beta_3\,\gamma_1}\left(\frac{\partial c}{\partial\beta}\right) > 0 & \frac{\partial\lambda_\pi}{\partial\beta} = -\alpha_1\frac{1-\beta_3\,\gamma_2}{\beta_2+\beta_3\,\gamma_1}\left(\frac{\partial c}{\partial\beta}\right) > 0
\end{array}
$$

### 6.7 Appendix 7: Dynamics of parameters

$$\frac{\partial c}{\partial\mu} = \frac{\alpha_1\left(-\mu+3\,\beta\,\mu+\beta\,\alpha_1{}^2+\sqrt{4\,\beta^2\,\mu\,\alpha_1{}^2+((-1+\beta)\,\mu+\beta\,\alpha_1{}^2)^2}\right)}{2\,\mu^2\,\sqrt{4\,\beta^2\,\mu\,\alpha_1{}^2+((-1+\beta)\,\mu+\beta\,\alpha_1{}^2)^2}}$$

$$\frac{\partial\lambda_\pi}{\partial\beta} = \frac{\mu-\beta\,\mu-\beta\,\alpha_1{}^2-\sqrt{4\,\beta^2\,\mu\,\alpha_1{}^2+((-1+\beta)\,\mu+\beta\,\alpha_1{}^2)^2}}{2\,\beta^2\,\alpha_1\,\sqrt{4\,\beta^2\,\mu\,\alpha_1{}^2+((-1+\beta)\,\mu+\beta\,\alpha_1{}^2)^2}}$$

$$\frac{\partial c}{\partial\alpha_1} = \frac{\left((-1+\beta)\,\mu-\beta\,\alpha_1{}^2\right)\left(\mu-\beta\,\mu-\beta\,\alpha_1{}^2+\sqrt{4\,\beta^2\,\mu\,\alpha_1{}^2+((-1+\beta)\,\mu+\beta\,\alpha_1{}^2)^2}\right)}{2\,\beta\,\mu\,\alpha_1{}^2\,\sqrt{4\,\beta^2\,\mu\,\alpha_1{}^2+((-1+\beta)\,\mu+\beta\,\alpha_1{}^2)^2}}$$

Under certain conditions, $\frac{\partial c}{\partial\mu} > 0$ (if $\mu < 3\,\beta\,\mu+\beta\,\alpha_1{}^2+\sqrt{4\,\beta^2\,\mu\,\alpha_1{}^2+((-1+\beta)\;\mu+\beta\,\alpha_1{}^2)^2}$), $\frac{\partial c}{\partial\beta} < 0$ (if $\mu < \beta\,\mu+\beta\,\alpha_1{}^2+\sqrt{4\,\beta^2\,\mu\,\alpha_1{}^2+((-1+\beta)\;\mu+\beta\,\alpha_1{}^2)^2}$), and $\frac{\partial c}{\partial\alpha_1} < 0$ (if $-\mu+\beta\,\mu+\beta\,\alpha_1{}^2+\sqrt{4\,\beta^2\,\mu\,\alpha_1{}^2+((-1+\beta)\;\mu+\beta\,\alpha_1{}^2)^2}$). $\frac{\partial c}{\partial\mu} > 0$, $\frac{\partial c}{\partial\beta} < 0$, and $\frac{\partial c}{\partial\alpha_1} < 0$.

### 6.8 Appendix 8: Taylor rule coefficients of the two-country model

With the same methodology than the basic model, we obtain the following definitions of the coefficients of $E_t\,[y_{t+1}]$ and $E_t\left[y^*_{t+1}\right]$:

$$
\begin{array}{ll}
\delta_1 = \frac{\beta_1+\alpha_1(\beta_2+\beta_3\gamma_1)}{1-\beta_3\gamma_2} & \delta_4 = \frac{\beta_5+\alpha_2(\beta_6+\beta_7\gamma_3)}{1-\beta_7\gamma_4} \\
\delta_2 = \frac{\beta_2+\beta_3\gamma_1}{1-\beta_3\gamma_2} & \delta_5 = \frac{\beta_6+\beta_7\gamma_3}{1-\beta_7\gamma_4} \\
\delta_3 = \frac{\beta_4}{1-\beta_3\gamma_2} & \delta_6 = \frac{\beta_8}{1-\beta_7\gamma_4}
\end{array}
$$

Thus, we obtain the following definitions of $y_{t+1}$ and $y^*_{t+1}$:

$$y_{t+1} = (\beta_1+\alpha_1\,(\beta_2+\beta_3\gamma_1)+\beta_3\gamma_2\delta_1)\,y_t-(\beta_2+\beta_3\gamma_1+\beta_3\gamma_2\delta_2)\,(i_t-\pi_t)+(\beta_4+\beta_3\gamma_2\delta_3)\,y^*_t+\eta_{t+1}$$

$$y^*_{t+1} = (\beta_5+\alpha_2\,(\beta_6+\beta_7\gamma_3)+\beta_7\gamma_4\delta_4)\,y_t-(\beta_6+\beta_7\gamma_3+\beta_7\gamma_4\delta_5)\,(i_t-\pi_t)+(\beta_8+\beta_7\gamma_4\delta_6)\,y^*_t+\eta^*_{t+1}$$

which can be written as

$$y_{t+1} = Ay_t - B(i_t - \pi_t) + Cy_t^* + \eta_{t+1}$$

$$y_{t+1}^* = Dy_t^* - E(i_t - \pi_t^*) + Fy_t + \eta_{t+1}^*$$

where

$$\begin{array}{ll} A = \beta_1 + \alpha_1(\beta_2 + \beta_3\gamma_1) + \beta_3\gamma_2\delta_1 & D = \beta_5 + \alpha_2(\beta_6 + \beta_7\gamma_3) + \beta_7\gamma_4\delta_4 \\ B = \beta_2 + \beta_3\gamma_1 + \beta_3\gamma_2\delta_2 & E = \beta_6 + \beta_7\gamma_3 + \beta_7\gamma_4\delta_5 \\ C = \beta_4 + \beta_3\gamma_2\delta_3 & F = \beta_8 + \beta_7\gamma_4\delta_6 \end{array}$$

Then we can express the foreign output, $y_t^*$, as following:

$$y_t^* = \frac{1}{D}\left(y_{t+1}^* + E(i_t - \pi_t^*) - Fy_t - \eta_{t+1}^*\right)$$

that we include in the expression of the domestic output to obtain

$$y_{t+1} = Ay_t - B(i_t - \pi_t) + \frac{C}{D}\left(y_{t+1}^* + E(i_t - \pi_t^*) - Fy_t - \eta_{t+1}^*\right) + \eta_{t+1}$$

Thus, with $y_t^* = \sigma_y y_t + \xi_t$ and $r_t^* = i_t - \pi_t^* = \sigma_\pi(i_t - \pi_t) + \psi_t$, we have:

$$y_{t+1} = Ay_t - B(i_t - \pi_t) + \frac{C}{D}\left(\sigma_y y_t + \xi_t + E\sigma_\pi(i_t - \pi_t) + E\psi_t - Fy_t - \eta_{t+1}^*\right) + \eta_{t+1}$$

$$y_{t+1}\left(1 - \sigma_y\frac{C}{D}\right) = \left(A - \frac{CF}{D}\right)y_t - \left(B - \sigma_\pi\frac{CE}{D}\right)(i_t - \pi_t) + \frac{C}{D}\left(\xi_t + E\psi_t - \eta_{t+1}^*\right) + \eta_{t+1}$$

$$y_{t+1} = \left(\frac{AD - CF}{D - \sigma_y C}\right)y_t - \left(\frac{BD - \sigma_\pi CE}{D - \sigma_y C}\right)(i_t - \pi_t) + \varpi_{t+1}$$

where $\varpi_{t+1} = \frac{C\left(\xi_t + E\psi_t - \eta_{t+1}^*\right) + D\eta_{t+1}}{D - \sigma_y C}$ and $\varphi_t' = \left(\frac{AD-CF}{D-\sigma_y C}\right)y_t - \left(\frac{BD-\sigma_\pi CE}{D-\sigma_y C}\right)(i_t - \pi_t)$

By using the same methodology than the basic model resolution, the optimization gives us the following results:

$$c\kappa_t = \varphi_t'$$

$$c(\pi_t + \alpha_1 y_t) = \left(\frac{AD - CF}{D - \sigma_y C}\right)y_t - \left(\frac{BD - \sigma_\pi CE}{D - \sigma_y C}\right)(i_t - \pi_t)$$

Thus,

$$i_t = \left(1 - c\left(\frac{D - \sigma_y C}{BD - \sigma_\pi CE}\right)\right)\pi + \left(\frac{AD - CF - c\alpha_1(D - \sigma_y C)}{BD - \sigma_\pi CE}\right)y_t$$

Thus, $\lambda_\pi = 1 - c\left(\frac{D-\sigma_y C}{BD-\sigma_\pi CE}\right)$ and $\lambda_y = \frac{AD-CF+c\alpha_1(D-\sigma_y C)}{BD-\sigma_\pi CE}$

## 6.9 Appendix 9

## 6.10 Appendix 10

## 6.11 Appendix 11: Estimations of parameters

Equation (8)
Observations: 107
Estimated parameter: $\alpha_1$

| | |
|---|---|
| $R^2$ | 0.99883 |
| Adjusted $R^2$ | 0.99882 |
| S.E. of regression | 0.01379 |
| Prob(F-statistic) | 0.00084 |
| Mean dependent var | 1.32033 |
| S.D. dependent var | 0.40308 |
| Sum squared resid | 0.02015 |

Equation (9)
Observations: 107
Estimated parameter: $\alpha_2$

| | |
|---|---|
| $R^2$ | 0.99958 |
| Adjusted $R^2$ | 0.99958 |
| S.E. of regression | 0.00579 |
| Prob(F-statistic) | 0.00175 |
| Mean dependent var | 0.91732 |
| S.D. dependent var | 0.28387 |
| Sum squared resid | 0.00355 |

Equation (10)
Observations: 106
Estimated parameters: $\beta_1, \beta_2, \beta_3, \beta_4$

| | |
|---|---|
| $R^2$ | 0.99955 |
| Adjusted $R^2$ | 0.99953 |
| S.E. of regression | 0.01544 |
| Prob(F-statistic) | 0.00241 |
| Mean dependent var | 1.37525 |
| S.D. dependent var | 0.71681 |
| Sum squared resid | 0.02433 |

Equation (11)
Observations: 106
Estimated parameters: $\beta_5, \beta_6, \beta_7, \beta_8$

| | |
|---|---|
| $R^2$ | 0.99986 |
| Adjusted $R^2$ | 0.99985 |
| S.E. of regression | 0.00885 |
| Prob(F-statistic) | 0.00178 |
| Mean dependent var | 0.99914 |
| S.D. dependent var | 0.73448 |
| Sum squared resid | 0.00799 |

# References


[1] Ball, Laurence M. 1997. "Efficient Rules for Monetary Policy," *NBER Working Paper No. 5952.*

[2] Bernanke, Ben and Mark Gertler. 2001. "Should Central Banks Respond to Movements in Asset Prices ?,"*American Economic Review*, Papers and Proceedings, 91 (2), 253-257.

[3] Bernanke, Ben and Mark Gertler. 2000. "Monetary Policy and Asset Price Volatility," *NBER Working Paper No. 7559*.

[4] Cecchetti, Stephen G., Hans Genberg, John Lipsky, and Sushil Wadhwani. 2000. "Asset Prices and Central Bank Policy", *International Centre for Monetary and Banking Studies*, London.

[5] Conover Mitchell, Gerald R. Jensen and Robert R. Johnson .1999. "Monetary Environments and International Stock Returns", *Journal of Banking and Finance*, Vol. 23, No. 9.

[6] Fama, Eugene. 1981. "Stock Returns, Real Activity, Inflation and Money," *American Economic Review*, Vol. 71, No. 4, 545-65.

[7] Filardo, Andrew J. 2000. "Monetary Policy and Asset Prices," *Economic Review*, Federal Reserve of Kansas City, Fourth Quarter, 11-37.

[8] Fuhrer, Jeffrey. 1997. "The (Un)Importance of Forward Looking Behavior in Price Specifications", *Journal of Money, Credit and Banking*, Vol. 29, No. 3, 338-350.

[9] Fuhrer Jeffrey C. and Moore George R. 1995. "Monetary Policy Trade-offs and the Correlation between Nominal Interest Rates and Real Output", *American Economic Review*, Vol. 85, No. 1, pp. 219-239.

[10] Goodhart, Charles. 2001. "What Weight Should be Given to Asset Prices in the Measurement of Inflation ?" *Economic Journal*, Vol. 111, 335-356.

[11] Goodhart, Charles and Boris Hofmann. 2000. "Financial Variables and the Conduct of Monetary Policy," *Sveriges Riskbank Working Paper No. 12*.

[12] Stock James H, Mark W. Watson. 2001. "Forecasting Output and Inflation: The Role of Asset Prices," *NBER Working Paper No. 8180.*

[13] Svensson, Lars E.O. 1997. "Inflation forecast targeting: Implementing and monitoring inflation targets," *European Economic Review*, 41, 1111-1146.

[14] Svensson, Lars E.O. 2002. "What is Wrong with Taylor Rules? Using Judgment in Monetary Policy through Targeting Rules," *NBER Working Paper No. 9421.*

[15] Vickers, John. 1999. "Monetary Policy and Asset Prices," Lecture given at the Money, Macro, and Finance Group, 31st Annual Conference, Oxford University.

[16] Walsh Carl E. 1998. "Monetary Theory and Policy," The MIT Press, Cambridge, Massachusetts.