The problem of maximization of C_t for any given expenditure level

$$\int_0^1 P_t(i)\, C_t(i)\, di = Z_t \tag{1}$$

can be formalized by means of the Lagrangean

$$L_t = \left[\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}}\, di\right]^{\frac{\varepsilon}{\varepsilon-1}} - \delta_t \left(\int_0^1 P_t(i)\, C_t(i)\, di - Z_t\right) \tag{2}$$

For all $i \in [0,1]$, the associated first-order conditions are

$$C_t(i)^{-\frac{1}{\varepsilon}}\, C_t^{\frac{1}{\epsilon}} = \delta_t P_t(i) \tag{3}$$

Thus, for any two goods (i,j), $C_t(i) = C_t(j)\left(\frac{P_t(i)}{P_t(j)}\right)^{-\varepsilon}$ which can be substituted into the expression for consumption expenditures to yield

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} \frac{Z_t}{P_t} \tag{4}$$

for all $i \in [0,1]$.

The latter condition can then be substituted into the definition of C_t to obtain

$$\int_0^1 P_t(i)\, C_t(i)\, di = P_t C_t \tag{5}$$

Combining the two previous equations yields the demand schedule equation

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t \tag{6}$$