New Keynesian DSGE models

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This presentation does not necessarily reflect the views of the Bank of Israel

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Layout

- Introduction
- A baseline model
- An augmented model
- Conclusion
Detailed layout (1)

- Introduction
  - What is a model?
  - What is a DSGE model?
  - Why DSGE modeling?
  - Literature
- The baseline model
  - Households
  - Firms
  - Central bank
  - Equilibrium
  - The DSGE model
Detailed layout (2)

- The augmented model
  - Theory
  - Empirics
  - Practice
  - Results

- Conclusion
  - Interpretation
  - Critics
  - Answers
What is a model?

A theoretical (hypothetical) construct that represents a process by a number of variables and a set of quantitative, or logical, relationships between them.

Jennifer Ann Francis (Physicist)

A simplistic theoretical and/or empirical method explaining complicated processes.

Peter James Richerson (Biologist)

The best way to simply (wrongly) describe some workings of the economy.

Unknown Economist
What are the objectives?

- Provide an explanation (story) about a specific state.
- Determine what might happen in different scenarios.
- Determine what might happen at a future date.
- Prescribe new (economic) guidelines that will change future (economic) behaviors.
- Assist decision making processes: trading, investment, planning, resource allocations, economic policy etc.
- Provide logical defense to justify (economic) policies: national/political, organizational, household, etc.
The birth

- Lucas (1973) on imperfect information, and subsequent research, provided fertile ground for examining the centrality of nominal rigidities in macroeconomics.
- The real business cycle (RBC) revolution, inspired by the neoclassical paradigm, finds an explanation in productivity dynamics (Kydland and Prescott, 1982).
- Integrating nominal rigidities in the RBC model gave birth to what we know as the New Keynesian Dynamic Stochastic General Equilibrium (NK DSGE) model.
What is a DSGE model?

- DSGE models are dynamic, stochastic, and characterize the general equilibrium of the economy.
- They make three strategic modeling choices:
  1. The behavior of agents is formally derived from microfoundations (microfounded). Agents are assumed to behave **optimally** and **rationally**.
  2. The underlying economic environment is that of a competitive economy, but with a number of essential distortions added: nominal rigidities, monopoly power, information problems, etc.
  3. The model is thought and estimated as a system, rather than equation by equation.
The basic structure of a DSGE model

\[ Y = f(Y^e, i - \pi^e, ...) \]  
Demand shocks

\[ \pi = f(\pi^e, Y, ...) \]  
Supply shocks

\[ i = f(\pi - \pi^*, Y, ...) \]  
Monetary policy

Productivity shocks

Expectations

\[ Y^e, \pi^e \]
Why DSGE modeling?

- A DSGE model is a VAR model with optimization-based constraints (economic foundations).
  - You like VAR? You will like DSGE.

- Isolating micro-parameters in VAR modeling is quasi-impossible.
  - You should like DSGE.

- Structural **microfounded shocks** cannot be identified in VAR models.
  - Can you love DSGE?
What is a VAR?

- Basically, a VAR consists of a set of $K$ endogenous variables $Y_t = (y_{1t}, \ldots, y_{kt}, \ldots, y_{Kt})$ for $k = 1, \ldots K$.
- A VAR model describes the evolution of a set of, let’s say $K = 3$, endogenous variables over the same sample period as a linear function of only their past values.

\[
\begin{bmatrix}
1 & A_1 & 0 \\
0 & 1 & A_2 \\
A_3 & A_4 & 1
\end{bmatrix}
\begin{bmatrix}
\pi_t \\
y_t \\
i_t
\end{bmatrix}
= 
\begin{bmatrix}
B_1 & 0 & 0 \\
B_2 & 1 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\mathbb{E}_t[\pi_{t+1}] \\
\mathbb{E}_t[y_{t+1}] \\
\mathbb{E}_t[i_{t+1}]
\end{bmatrix}
+ 
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & C_1
\end{bmatrix}
\begin{bmatrix}
\pi_{t-1} \\
y_{t-1} \\
i_{t-1}
\end{bmatrix}
+ 
\begin{bmatrix}
D_1 \\
D_2 \\
D_3
\end{bmatrix}
+ 
\begin{bmatrix}
1 & E_1 & 0 \\
0 & 0 & 0 \\
0 & E_2 & 1
\end{bmatrix}
\begin{bmatrix}
\varepsilon_t^p \\
\varepsilon_t^a \\
\varepsilon_t^i
\end{bmatrix}
\]
Why comparing to VAR?

- It is the closest modeling methodology.
- It is, and was, well accepted by the literature.
- Data science to single equation methodologies are too far from DSGE theoretical models.
- Yet, too much theory kills model’s applications.
Why do we need DSGE?

- The economy is thought as an interlinked system, excluding single equation methodologies (OLS, IV etc).
- Although it is possible to build an empirical methodology to estimate an economy thought as a system (VAR, SUR etc.), only DSGE modeling allows for a clear assessment and identification of structural shocks and structural parameters.
- DSGE models also allow to simulate calibrated microfounded models (calibration can be base on the literature or a previous estimation) without any available data.
- Policy assessment based on microeconomic behaviors and interlinkages, with or without data, is easier through DSGE models.
- To date, and to my limited knowledge, there is no simpler alternative.
Warning

Remember, dreams without goals are just dreams and they ultimately fuel disappointment.

Denzel Washington (Actor)

- In other words, never build a model without a **clear research question**. *Have dreams, but have goals*...
- Before the research question, you should:
  - have a clear intuition about the ingredients of your economy...
    ...Occam’s razor.
  - think how ingredients should **interact**.
  - expect results (priors).
- DSGE models are not the only tool available to macroeconomists.
What is a simple DSGE?

- Two very simple models will be presented:
  - A baseline DSGE model (4 equations).
  - A simple extension (6 equations).

- Two requirements:
  - Basic mathematical knowledge: optimization, derivation, linearization.
  - Savlanout!
Selected books


Selected papers


Baseline framework

The model of Galí (2015) consists of economic agents of 3 types:

- **Households**
  Purchase goods for consumption, hold money and bonds, supply labor, and maximize the expected present value of utility.

- **Firms**
  Hire labor, produce and sell differentiated products in monopolistically competitive goods markets, and maximize profits.

- **Central bank**
  Controls the nominal rate of interest.
Households utility and budget constraint

- Preferences of the representative household are defined over a composite consumption good \( C_t \), and leisure \( 1 - N_t \), where \( N_t \) is the time devoted to market employment.

- Standard (CRRA) utility function

\[
U_t = \frac{1}{1 - \sigma} C_t^{1-\sigma} - \frac{\chi N_t^{1+\eta}}{1 + \eta}
\]  
(1)

- Budget constraint

\[
P_t C_t + Q_t B_t + M_t \leq B_{t-1} + W_t N_t + M_{t-1}
\]  
(2)
Solving the model

- **Budget constraint**

\[
C_t + Q_t \frac{B_t}{P_t} + \frac{M_t}{P_t} \leq \frac{B_{t-1}}{P_t} + \frac{W_t}{P_t} N_t + \frac{M_{t-1}}{P_t} \tag{3}
\]

- **Solvency condition**

\[
\forall t \lim_{n \to \infty} \mathbb{E}_t [B_n] \geq 0 \tag{4}
\]

- **Households’ Lagrangian**

\[
\mathbb{E}_t \left[ U_{t+k} - \delta_{t+k} \left( C_{t+k} + \frac{M_{t+k}}{P_{t+k}} + Q_{t+k} \frac{B_{t+k}}{P_{t+k}} \right) \right] - \frac{B_{t-1+k}}{P_{t+k}} - \frac{W_{t+k}}{P_{t+k}} N_{t+k} - \frac{M_{t-1+k}}{P_{t+k}}
\]
Optimization (1)

Consumption, labor supply, and bond holdings are chosen to maximize $L_t$ subject to the budget constraint and the solvency condition:

$$\frac{\partial L_t}{\partial C_t} = 0 \implies \delta_t = C_t^{-\sigma}$$  \hspace{1cm} (5)

$$\frac{\partial L_t}{\partial B_t} = 0 \implies Q_t = \beta \mathbb{E}_t \left[ \frac{\delta_{t+1}}{\delta_t} \frac{P_t}{P_{t+1}} \right]$$  \hspace{1cm} (6)

$$\frac{\partial L_t}{\partial N_t} = 0 \implies \chi N_t^{\eta} = \delta_t \frac{W_t}{P_t}$$  \hspace{1cm} (7)
Optimization (2)

- Assumption: existence of a continuum of goods represented, or indexed, by \( i \in [0, 1] \).
- \( C_t \) is a consumption index given by

\[
C_t = \left( \int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}}
\]  

(8)

where \( C_t(i) \) represents the quantity of good \( i \) consumed by the household in period \( t \), and \( \varepsilon \) the elasticity of substitution between differentiated goods (of the same origin).

- The household must decide how to allocate its consumption expenditures among the different goods.
- This requires maximization of the consumption index, \( C_t \), for any given level of expenditure, i.e.

\[
\int_0^1 P_t(i) \, C_t(i) \, di = P_t C_t
\]  

(9)
Optimization (3)

The solution to that problem yields the set of demand equation

$$C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t$$

(10)

for all $i \in [0, 1]$, where

$$P_t = \left[ \int_0^1 P_t(i)^{1-\epsilon} \, di \right]^{\frac{1}{1-\epsilon}}$$

(11)

is an aggregate price index.
Production

- Assumption: existence of a continuum of firms represented, or indexed, by $i \in [0, 1]$
- Each firm produces a differentiated good, but they all use an identical technology, represented by the production function

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \quad (12)$$

where $A_t = \exp(\varepsilon^a_t)$ represents the level of technology.
- Assumption: $\varepsilon^a_t$ is the technology shock common to all firms, evolving exogenously over time.
Demand and prices

► All firms face
  ► an identical isoelastic demand schedule (Eq. 10),
  ► and take $P_t$ (aggregate price level) and $C_t$ (aggregate consumption index) as given.

► Following Calvo (1983), each firm may reset its price only with probability $1 - \theta$ in any given period, independent of the time elapsed since the last adjustment.

► For each period a measure $1 - \theta$ of producers reset their prices, while a fraction $\theta$ keep their prices unchanged.\(^1\)

► In this context, $\theta$ becomes a natural index of price stickiness.

\(^1\)The average duration of a price is given by $(1 - \theta)^{-1}$
Price dynamics (1)

- Let $S(t) \subset [0, 1]$ represents the set of firms not reoptimizing their posted price in period $t$.
- Using the definition of the aggregate price level and the fact that all firms resetting prices will choose an identical price $P^*_t$,

$$P_t = \left[ \int_{S(t)} P_{t-1}(i)^{1-\varepsilon} \, di + (1 - \theta) (P^*_t)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$$

$$= \left[ \theta P_{t-1}^{1-\varepsilon} + (1 - \theta) (P^*_t)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$$  \hspace{1cm} (13)

where the second equality follows from the fact that the distribution of prices among firms not adjusting in period $t$ corresponds to the distribution of effective prices in period $t - 1$, though with total mass reduced to $\theta$. 


Price dynamics (2)

- Dividing both sides by $P_{t-1}$,

$$
\Pi_t^{1-\varepsilon} = \theta + (1 - \theta) \left( \frac{P_t}{P_{t-1}} \right)^{1-\varepsilon}
$$

(14)

where $\Pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate between $t - 1$ and $t$.

- Log-linearization of the above equation around $\Pi_t = 1$ and $\frac{P_t}{P_{t-1}} = 1$ yields

$$
\pi_t = (1 - \theta) (p_t^* - p_{t-1})
$$

(15)

- Inflation results from the fact that firms reoptimizing in any given period choose a price that differs from the economy’s average price in the previous period.
Price setting (1)

- A firm reoptimizing in period $t$ will choose the price $P_t^*$ that maximizes the current market value of the profits generated while that price remains effective, i.e.

$$\max_{P_t^*} \left\{ \sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left[ Q_{t,t+k} \left( P_t^* Y_{t+k|t} - \Psi_{t+k} (Y_{t+k|t}) \right) \right] \right\}$$

subject to the sequence of demand constraints

$$Y_{t+k|t} = \left( \frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} C_{t+k} \quad \text{for} \quad k = 0, 1, 2, \ldots \text{ where}$$

$$Q_{t,t+k} = \beta^k \left( \frac{C_{t+k}}{C_t} \right)^{-\sigma} \frac{P_t^*}{P_{t+k}} \quad \text{is the stochastic discount factor} \quad \text{for nominal payoffs,} \quad \Psi(\cdot) \quad \text{is the cost function, and} \quad Y_{t+k|t} \text{ denotes output in period} \quad t + k \quad \text{for a firm that last reset its price in period} \quad t.$$
Price setting (2)

The first-order condition associated with the above problem takes the form

$$\sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left[ Q_{t,t+k} Y_{t+k|t} \left( P_t^* - \frac{\varepsilon}{\varepsilon - 1} \Psi'_{t+k} (Y_{t+k|t}) \right) \right] = 0$$  \hspace{1cm} (17)

where $\Psi'_{t+k} (Y_{t+k|t})$ denotes the (nominal) marginal cost in period $t + k$ for a firm which last reset its price in period $t$.

Note that in the limiting case of no price rigidities ($\theta = 0$), the previous condition collapses to the familiar optimal price-setting condition under flexible prices

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \Psi'_{t+k} (Y_{t+k|t})$$  \hspace{1cm} (18)

which allows us to interpret $\frac{\varepsilon}{\varepsilon - 1}$ as the desired markup in the absence of constraints on the frequency of price adjustment.
Price setting (3)

Next, the optimal price-setting condition (Eq. 17) is linearized around the zero inflation steady state.

Rewriting Eq. 17 in terms of variables that have a well-defined value in that steady state\(^2\) leads to

\[
\sum_{k=0}^{\infty} \theta^k E_t \left[ Q_{t,t+k} Y_{t+k|t} \left( \frac{P^*_t}{P_{t-1}} - \frac{\varepsilon}{\varepsilon - 1} MC_{t+k|t} \Pi_{t-1,t+k} \right) \right] = 0
\]

(19)

where \( MC_{t+k|t} = \frac{\Psi'_{t+k}(Y_{t+k|t})}{P_{t+k}} \) is the real marginal cost in period \( t + k \) for a firm whose price was last set in period \( t \).

In the zero inflation steady state, \( \frac{P^*_t}{P_{t-1}} = 1 \) and \( \Pi_{t-1,t+k} = 1 \).

\(^2\)Dividing by \( P_{t-1} \) and letting \( \Pi_{t,t+k} = \frac{P_{t+k}}{P_t} \).
Price setting (4)

- Furthermore, $P^*_t = P_{t+k}$ in that steady state, from which it follows that $Y_{t+k|t} = Y$ and $MC_{t+k|t} = MC$, because all firms will be producing the same quantity of output.

- In addition, $Q_{t,t+k} = \beta^k$ must hold in that steady state, and $MC = \frac{\varepsilon - 1}{\varepsilon}$.

- A first-order Taylor expansion of Eq. 19 around the zero inflation steady state yields to

$$p^*_t - p_{t-1} = (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k \mathbb{E}_t \left[ mc_{t+k|t} + (p_{t+k} - p_{t-1}) \right]$$

where $mc_{t+k|t} = mc_{t+k|t} - mc$ denotes the log deviation of marginal cost from its steady state value $mc = -\mu$, and $\mu = \ln \left( \frac{\varepsilon}{\varepsilon - 1} \right)$ is the log of the desired gross markup.
Goods market

- Market clearing in the goods market requires

\[ Y_t(i) = C_t(i) \]  \hspace{1cm} (21)

for all \( i \in [0, 1] \) and all \( t \).

- Letting aggregate output be defined as

\[ Y_t = \left( \int_0^1 Y_t(i)^{1-\frac{1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}} \]  \hspace{1cm} (22)

it follows that \( Y_t = C_t \) must hold for all \( t \).

- One can combine the above goods market clearing condition with the consumer’s Euler equation (Eq. 6) to obtain the equilibrium condition

\[ y_t = \mathbb{E}_t [y_{t+1}] - \sigma^{-1} (i_t - \mathbb{E}_t [\pi_{t+1}] - \rho) \]  \hspace{1cm} (23)
Labor market

- Market clearing in the labor market requires

\[ N_t = \int_0^1 N_t(i) \, di \quad (24) \]

- Using the production function (Eq. 12) leads to

\[
N_t = \int_0^1 \left( \frac{Y_t(i)}{A_t} \right)^{\frac{1}{1-\alpha}} \, di \\
= \left( \frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\frac{\epsilon}{1-\alpha}} \, di \quad (25)
\]

where the second equality follows from the demand schedule (Eq. 10) and the goods market clearing condition.
Taking logs,\(^3\)
\[(1 - \alpha) n_t = y_t - \varepsilon_t^a + d_t\] (26)

One can write the following approximate relation between aggregate output, employment, and technology as
\[y_t = \varepsilon_t^a + (1 - \alpha) n_t\] (27)

\(^3d_t\) is equal to zero up to a first-order approximation.
Marginal cost (2)

An expression is derived for an individual firm’s marginal cost in terms of the economy’s average real marginal cost

\[ mc_t = (w_t - p_t) - mpn_t \]
\[ = (w_t - p_t) - (\varepsilon_t^a - \alpha n_t) - \ln (1 - \alpha) \quad (28) \]
\[ = (w_t - p_t) - \frac{1}{1 - \alpha} (\varepsilon_t^a - \alpha y_t) - \ln (1 - \alpha) \quad (29) \]

where Eq. 28 defines the economy’s average marginal product of labor, \( mpn_t \), in a way consistent with Eq. 27.
Marginal cost (3)

Using the fact that

\[
mc_{t+k|t} = (w_{t+k} - p_{t+k}) - mpn_{t+k|t}
\]

\[
= (w_{t+k} - p_{t+k}) - \frac{1}{1 - \alpha} (\varepsilon_{t+k}^a - \alpha y_{t+k|t}) - \ln (1 - \alpha)
\]

then

\[
mc_{t+k|t} = mc_{t+k} + \frac{\alpha}{1 - \alpha} (y_{t+k|t} - y_{t+k})
\]

\[
= mc_{t+k} - \frac{\alpha \varepsilon}{1 - \alpha} (p^*_t - p_{t+k})
\]

where the second equality follows from the demand schedule (Eq. 10) combined with the market clearing condition \((c_t = y_t)\).
Phillips curve (1)

Substituting Eq. 30 into Eq. 20 and rearranging terms yields

\[ p_t^* - p_{t-1} = (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k \mathbb{E}_t [\Theta \hat{m}c_{t+k} + (p_{t+k} - p_{t-1})] \]

\[ = (1 - \beta \theta) \Theta \sum_{k=0}^{\infty} (\beta \theta)^k \mathbb{E}_t [\hat{m}c_{t+k}] + \sum_{k=0}^{\infty} (\beta \theta)^k \mathbb{E}_t [\pi_{t+k}] \]

where \( \Theta = \frac{1-\alpha}{1-\alpha+\alpha \varepsilon} \leq 1. \)

Notice that the above discounted sum can be rewritten more compactly as the difference equation

\[ p_t^* - p_{t-1} = \beta \theta \mathbb{E}_t [p_{t+1}^* - p_t] + (1 - \beta \theta) \Theta \hat{m}c_t + \pi_t \quad (31) \]
Phillips curve (2)

Finally, combining Eq. 15 and Eq. 31 yields to the inflation equation

\[ \pi_t = \beta E_t [\pi_{t+1}] + \lambda \hat{m} c_t \]  \hspace{1cm} (32)

where \( \lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta} \Theta \) is strictly decreasing\(^4\) in

- the index of price stickiness \( \theta \),
- the measure of decreasing returns \( \alpha \),
- and in the demand elasticity \( \varepsilon \).

\(^4\) Note that \( \theta < 1 \), \( \beta < 1 \), \( \alpha < 1 \), and \( \varepsilon > 1 \).
Phillips curve (3)

- A relation is derived between the economy’s real marginal cost and a measure of aggregate economic activity

\[
mc_t = (w_t - p_t) - m\pi_t
\]
\[
= (\sigma y_t + \varphi n_t) - (y_t - n_t) - \ln (1 - \alpha)
\]
\[
= \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) y_t - \frac{1 + \varphi}{1 - \alpha} \epsilon_t^{a} - \ln (1 - \alpha)
\]

where derivation of Eq. 33 and Eq. 34 make use of:

- the household’s optimality condition (Eq. 7),
- and the (approximate) aggregate production relation (Eq. 27).
Phillips curve (4)

- Under flexible prices the real marginal cost is constant and given by $mc = -\mu$.
- Defining the natural level of output, denoted by $y_t^f$, as the equilibrium level of output under flexible prices

$$mc = \left( \sigma + \frac{\phi + \alpha}{1 - \alpha} \right) y_t^f - \frac{1 + \phi}{1 - \alpha} \varepsilon_t^a - \ln (1 - \alpha) \quad (35)$$

implies

$$y_t^f = \psi^f \varepsilon_t^a + v_t^f \quad (36)$$

where $v_t^f = -\frac{(1 - \alpha)(\mu - \ln(1 - \alpha))}{\sigma(1 - \alpha) + \phi + \alpha} > 0$ and $\psi^f = \frac{1 + \phi}{\sigma(1 - \alpha) + \phi + \alpha}$.

- Notice that when $\mu = 0$ (perfect competition), the natural level of output corresponds to the equilibrium level of output in the classical economy.
Phillips curve (5)

- Subtracting Eq. 35 from Eq. 34 leads to
  \[
  \hat{mc}_t = mc_t - mc = \left( \sigma + \frac{\phi + \alpha}{1 - \alpha} \right) \left( y_t - y_t^f \right)
  \]  
  (37)

- By combining Eq. 37 with Eq. 32 one can obtain an equation relating inflation to its one period ahead forecast and the output gap
  \[
  \pi_t = \beta \mathbb{E}_t \left[ \pi_{t+1} \right] + \kappa \left( y_t - y_t^f \right)
  \]  
  (38)

where \( \kappa = \lambda \left( \sigma + \frac{\phi + \alpha}{1 - \alpha} \right) \).

- Eq. 38 is often referred to as the New Keynesian Phillips curve and constitutes one of the key building blocks of the basic New Keynesian model.

- The second key equation describing the equilibrium of the New Keynesian model is the Euler equation (Eq. 23).
Closing the model

- We add an ad-hoc monetary policy rule to close the model (Taylor-type rule).
- We can add a demand shock or a cost-push shock, but not both: 3 shocks because 3 historical variables.
- Structural shocks are assumed to follow a first-order autoregressive process with an \( i.i.d. \)-normal error term such as

\[
\varepsilon^k_t = \rho_k \varepsilon^k_{t-1} + \omega_{k,t} \quad \text{where} \quad \varepsilon_{k,t} \sim N(0; \sigma_k) \quad \text{for} \quad k = \{p, i, a\}
\]
Not fully microfounded

\[ \pi_t = \beta \mathbb{E}_t [\pi_{t+1}] + \kappa \left( y_t - y_t^f \right) + \varepsilon_t^p \]  \hspace{1cm} (39)

\[ y_t = \mathbb{E}_t [y_{t+1}] - \sigma^{-1} (i_t - \mathbb{E}_t [\pi_{t+1}] - \rho) \]  \hspace{1cm} (40)

\[ y_t^f = \psi^f \varepsilon_t^a + \upsilon^f \]  \hspace{1cm} (41)

\[ i_t = (1 - \lambda_i) \left( \lambda_\pi \pi_t + \lambda_x (y_t - y_t^f) \right) + \lambda_i i_{t-1} + \varepsilon_t^i \]  \hspace{1cm} (42)

Here, three shocks, two are ad-hoc and one is microfounded: a monetary policy shock (\(\varepsilon_t^i\)), a technology shock (\(\varepsilon_t^a\)) and a cost-push shock (\(\varepsilon_t^p\)).
Remember VAR models

\[
\begin{bmatrix}
1 & -\kappa & 0 \\
0 & 1 & 1/\sigma \\
-\lambda \pi (1 - \lambda_i) & -\lambda_x / \sigma & 1
\end{bmatrix}
\begin{bmatrix}
\pi_t \\
y_t \\
i_t
\end{bmatrix}
= 
\begin{bmatrix}
\beta & 0 & 0 \\
-1/\sigma & 1 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\mathbb{E}_t[\pi_{t+1}] \\
\mathbb{E}_t[y_{t+1}] \\
\mathbb{E}_t[i_{t+1}]
\end{bmatrix}
+ 
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & \lambda_i \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\pi_{t-1} \\
y_{t-1} \\
i_{t-1}
\end{bmatrix}
+ 
\begin{bmatrix}
-\kappa \nu^n \\
-\sigma^{-1} \rho \\
-\lambda_x \sigma^{-1} \nu^n
\end{bmatrix}
+ 
\begin{bmatrix}
1 & -\kappa \psi^n & 0 \\
0 & 0 & 0 \\
0 & -\lambda_x \sigma^{-1} \psi^n & 1
\end{bmatrix}
\begin{bmatrix}
\varepsilon^p_t \\
\varepsilon^a_t \\
\varepsilon^i_t
\end{bmatrix}
\]
Intuition

- Existing literature almost forgot the link between consumption and money.
- Yet, risk aversion impacts the trade-off between holding money and consuming in real life.
- If you take this link into account, should the central bank react with respect to a monetary aggregate?
Specific literature


Households utility and budget constraint

- Preferences of the representative household are defined over a composite consumption good \( C_t \), real money balances \( \frac{M_t}{P_t} \), and leisure \( 1 - N_t \).
- CES MIU utility function

\[
U_t = \frac{1}{1 - \sigma} \left( (1 - b) C_t^{1-\nu} + b e^{e_t^m} \left( \frac{M_t}{P_t} \right)^{1-\nu} \right)^{\frac{1-\sigma}{1-\nu}} - \frac{\chi N_t^{1+\eta}}{1 + \eta}
\]  

(43)

- The budget constraint (Eq. 3) and the solvency condition (Eq. 4) are identical.
- First order conditions will be very different from Eq. 5, Eq. 6 and Eq. 7.
First order conditions (1)

- The FOC corresponding to the demand for contingent bonds implies that

\[
\hat{c}_t = \mathbb{E}_t [\hat{c}_{t+1}] - (\hat{i}_t - \mathbb{E}_t [\hat{\pi}_{t+1}]) / (\nu - a_1 (\nu - \sigma)) \\
- \frac{(1 - a_1) (\nu - \sigma)}{\nu - a_1 (\nu - \sigma)} \mathbb{E}_t [\Delta \hat{m}_{t+1} - \hat{\pi}_{t+1}] + \zeta_{t,c} \tag{44}
\]

where \( \zeta_{t,c} = - \frac{(1-a_1)(\nu-\sigma)}{(1-\nu)(\nu-a_1(\nu-\sigma))} \mathbb{E}_t [\Delta \varepsilon^m_{t+1}] \) and by using the steady state of the first order conditions

\( a_1^{-1} = 1 + \left( \frac{b}{1-b} \right)^{\frac{1}{\nu}} (1 - \beta)^{\frac{\nu-1}{\nu}}. \)

- The lowercase (\( \hat{\cdot} \)) denotes the log-linearized (around the steady state) form of the original aggregated variables.
First order conditions (2)

- The FOC corresponding to the optimal demand for cash is given by
  \[-\nu (\hat{m}_t - \hat{p}_t) + \nu \hat{c}_t + \varepsilon^m_t = a_2 \hat{t}_t \quad (45)\]
  with \(a_2 = \frac{1}{\exp(1/\beta)-1}\) and where real cash holdings depend positively on consumption with an elasticity equal to unity and negatively on the nominal interest rate.

- The FOC corresponding to the optimal consumption-leisure arbitrage implies that
  \[\hat{w}_t - \hat{p}_t = \eta \hat{n}_t + (\nu - a_1 (\nu - \sigma)) \hat{c}_t \]
  \[- (\nu - \sigma) (1 - a_1) (\hat{m}_t - \hat{p}_t) + \zeta_{t,m} \quad (46)\]

where \(\zeta_{t,m} = -\frac{(\nu - \sigma)(1 - a_1)}{1 - \nu} \varepsilon^m_t\).
First order conditions (3)

- Finally, these equations represent:
  - the Euler condition for the optimal intratemporal allocation of consumption (Eq. 44),
  - the intertemporal optimality condition setting the marginal rate of substitution between money and consumption equal to the opportunity cost of holding money (Eq. 45),
  - and the intratemporal optimality condition setting the marginal rate of substitution between leisure and consumption equal to the real wage (Eq. 46).
Production function and prices

- Let’s use the same production function (Eq. 12)
- Here, we consider a microfounded cost-push shock, called price-markup shock, such as

\[ C_t = \left( \int_0^1 C_t(i)^{1-\frac{1}{\Lambda_t}} \, di \right)^{\frac{\Lambda_t}{\Lambda_t-1}} \tag{47} \]

where \( \Lambda_t = 1 + \frac{1}{\varepsilon - 1 + \varepsilon_t} \) is the elasticity of substitution between consumption goods in period \( t \).
- \( \frac{\Lambda_t}{\Lambda_t-1} \) is the (time varying) markup of prices over marginal costs.
- We will see how and why this feature can be considered as a price markup shock.
Price-markup shock

- Instead of Eq. 32, we obtain a time varying $\Theta$, that is now $\Theta_t = \frac{1-\alpha}{1-\alpha+\alpha \Lambda_t} \leq 1$, which take into account the price-markup shock.

- The inflation equation becomes

$$\pi_t = \beta \mathbb{E}_t [\pi_{t+1}] + \lambda_{mc_t} \hat{m}_c$$

where $\lambda_{mc_t} = \frac{(1-\theta)(1-\beta\theta)}{\theta} \Theta_t$. 


Solving the model (1)

- In addition to the previous Phillips curve (Eq. 48), we used Lagrangian method to optimize the utility function with respect to the budget constraint, and obtained the three first-order optimal conditions.
- We log-linearize around the steady state these conditions (Eq. 44, Eq. 45, and Eq. 46).
- We add an ad-hoc Taylor type rule equation to close our model.
Solving the model (2)

- Finally, we have 6 equations of 6 unknown variables for our economy:
  - the output gap ($\hat{y}_t$) and its flexible-price counterpart ($\hat{y}_t^f$),
  - the real money balances ($\hat{m}_t$) and its flexible-price counterpart ($\hat{m}_t^f$),
  - the inflation rate ($\hat{\pi}_t$),
  - and the nominal interest rate ($\hat{i}_t$).

- Structural shocks are also assumed to follow a first-order autoregressive process with an \textit{i.i.d.}-normal error term such as
  $$\varepsilon_t^k = \rho_k \varepsilon_{t-1}^k + \omega_{k,t} \text{ where } \varepsilon_{k,t} \sim N(0; \sigma_k) \text{ for } k = \{p, m, i, a\}.$$
The DSGE model

\[ \hat{y}_t^f = v^y_a \varepsilon_t^a + v^y_m \hat{m}p_t - v^y_c + v^y_{sm} \varepsilon_t^m \]  
(49)

\[ \hat{m}p_t^f = v^m_{y+1} \mathbb{E}_t [\hat{y}_t^{f+1}] + v^m_y \hat{y}_t^f + \frac{1}{v} \varepsilon_t^m \]  
(50)

\[ \hat{\pi}_t = \beta \mathbb{E}_t [\hat{\pi}_{t+1}] + \kappa_{x,t} (\hat{y}_t - \hat{y}_t^f) + \kappa_{m,t} (\hat{m}p_t - \hat{m}p_t^f) \]  
(51)

\[ \hat{y}_t = \mathbb{E}_t [\hat{y}_{t+1}] - \kappa_r (\hat{i}_t - \mathbb{E}_t [\hat{\pi}_{t+1}]) + \kappa_{mp} \mathbb{E}_t [\Delta \hat{m}p_{t+1}] + \kappa_{sm} \mathbb{E}_t [\Delta \varepsilon_{t+1}^m] \]  
(52)

\[ \hat{m}p_t = \hat{y}_t - \kappa_i \hat{i}_t + \frac{1}{v} \varepsilon_t^m \]  
(53)

\[ \hat{i}_t = (1 - \lambda_i) \left( \lambda_{\pi} (\hat{\pi}_t - \pi_c) + \lambda_x (\hat{y}_t - \hat{y}_t^f) + \lambda_m \tilde{M}_{t,k} \right) \]
Microfounded model

\[
\begin{align*}
\nu_a^y &= \frac{1+\eta}{(v-(v-\sigma)a_1)(1-\alpha)+\eta+\alpha} \\
\nu_m^y &= \frac{(1-\alpha)(v-\sigma)(1-a_1)}{(v-(v-\sigma)a_1)(1-\alpha)+\eta+\alpha} \\
\nu_c^y &= \frac{(1-\alpha)}{(v-(v-\sigma)a_1)(1-\alpha)+\eta+\alpha} \ln \left( \frac{\varepsilon}{\varepsilon-1} \right) \\
\nu_{sm}^m &= \frac{(v-(v-\sigma)a_1)(1-\alpha)+\eta+\alpha}{(v-\sigma)(1-a_1)(1-\alpha)} \\
v_{y+1}^m &= -\frac{a_2}{v} (v - (v - \sigma) a_1) \\
v_{y}^m &= 1 + \frac{a_2}{v} (v - (v - \sigma) a_1) \\
k_m,t &= (\sigma - v) (1 - a_1) \frac{(1-\alpha)\left(\frac{1}{\theta} - \beta\right)(1-\theta)(1+(\varepsilon-1)\varepsilon_t^p)}{1+(\alpha+\varepsilon_t^p)(\varepsilon-1)} \\
k_x,t &= \left( v - (v - \sigma) a_1 + \frac{\eta+\alpha}{1-\alpha} \right) \frac{(1-\alpha)\left(\frac{1}{\theta} - \beta\right)(1-\theta)(1+(\varepsilon-1)\varepsilon_t^p)}{1+(\alpha+\varepsilon_t^p)(\varepsilon-1)} \\
k_r &= \frac{1}{v-a_1(v-\sigma)} \\
k_{mp} &= \frac{(\sigma-v)(1-a_1)}{v-a_1(v-\sigma)} \\
k_i &= a_2 / v \\
k_{sm} &= -\frac{(1-a_1)(v-\sigma)}{(v-a_1(v-\sigma))(1-v)} \\
a_1 &= \frac{1}{1+(b/(1-b))^{1/v}(1-\beta)(\nu-1)/\nu} \\
a_2 &= \frac{1}{\exp(1/\beta)-1}
\end{align*}
\]
Money in the Taylor rule

- To evaluate further the role of money we analyze different specifications of the Taylor rule ($\tilde{M}_{t,k}$ for $k = \{0, 1, 2, 3\}$).
- We test four types of Taylor rules:
  - without money ($\tilde{M}_{t,0} = 0$).
  - with a real money gap ($\tilde{M}_{t,1} = \hat{m}p_t - \hat{m}p_t^f$).
  - with a nominal money growth ($\tilde{M}_{t,2} = \hat{m}t - \hat{m}t_{t-1}$).
  - with a real money growth ($\tilde{M}_{t,3} = \hat{m}p_t - \hat{m}p_{t-1}$).
Why not least squares or simple FIML?

- Impossible to capture inter-equations dynamics.
- Large sample size is needed.
- Unable to estimate micro-parameters.
- Unable to capture microfounded shocks role.
Why not GMM, doubles least squares or other IV estimators?

- How to select optimally IV?
- Large sample size is needed.
- Unable to estimate micro-parameters.
- Unable to capture microfounded shocks role.
Why not maximum likelihood?

- An and Schorfheide (2007).
- Identification issues for misspecified parameters.
- Large sample size are needed.
Why not VAR / BVAR?

- An and Schorfheide (2007).
- Unable to capture microfounded shocks role.
Why Bayesian estimation of DSGE models?

- Likelihood of the model (in terms of interlinked equation and interlinked macro-parameters).
- Able to study all micro-parameters and microfounded shocks.
- Less historical data are needed (but more reliance on priors).
Empirical methodology

- As in Smets and Wouters (2003) and An and Schorfheide (2007), we apply Bayesian techniques to estimate our DSGE model.
- We use Eurozone data like Andrès et al. (2006) and Barthélemy, Clerc and Marx (2011) from the Euro Area Wide Model (AWM) database (Fagan, Henry and Mestre, 2001).
- We use the M3 monetary aggregate from the Eurostat database, which is the broadest monetary aggregate.
- To make output and real money balances stationary, we use first detrended data, as in Ireland (2004), Andrés, López-Salido and Vallés (2006), and Barthélemy, Clerc and Marx (2011).
Data

- $\hat{\pi}_t$ is the log-linearized detrended inflation rate measured as the yearly log difference of detrended GDP Deflator from one quarter to the same quarter of the previous year;
- $\hat{y}_t$ is the log-linearized detrended output per capita measured as the difference between the log of the real GDP per capita and its trend;
- $\hat{i}_t$ is the 3-month detrended nominal interest rate.
- $\hat{mp}_t$ is the log-linearized detrended real money balances per capita measured as the difference between the real money per capita (log difference between the money stock per capita and the GDP Deflator) and its trend.
- $\hat{y}_t^f$, the flexible-price output, and $\hat{mp}_t^f$, the flexible-price real money balances, are completely determined by structural shocks.
Calibration

- Following standard conventions, we calibrate **beta** distributions for parameters that fall between zero and one, **inverted gamma** distributions for parameters that need to be constrained to be greater than zero, and **normal** distributions in other cases.

- The calibration of $\sigma$ is inspired by Rabanal and Rubio-Ramírez (2007) and Casares (2007), respectively of 2.5 and 1.5.

- $\sigma = 2$ corresponds to a standard risk aversion.

- $\sigma = 4$, represents a high level of risk aversion.\(^5\)

- As our goal is to analyze two different configurations of risk, we adopt the same priors in the two models with different risk aversion calibrations.

\(^5\)Twice the standard value. $\sigma = 4$ is also around twice the estimated value in normal periods.
Estimation characteristics

- Sample: 117 observations from 1980Q4 to 2009Q4 in order to avoid high volatility periods before 1980.
- Algorithm: Metropolis-Hastings of 10 distinct chains, each of 100,000 draws (Smets and Wouters, 2007; Adolfson et al., 2007).
- Average acceptation rate per chain for the benchmark model ($\sigma$ estimated) are included in the interval [0.2601; 0.2661] and for ($\sigma = 4$) in the interval [0.2587; 0.2658].
Dynare (1)

```
var y, pi, r, mp, yf, mpf, ep, ei, em, at;
varexo up, ui, um, ua;
parameters alpha beta teta vega sigma neta epsilon b a1 a2 li1 li2 li3 li4 rhoa rhop rholi rhoi rhom pb yb mpb rb;
alpha = 0.33;
beta = 0.99;
teta = 0.66;
vega = 1.25;
sigma = 2.0;
b = 0.25;
a1 = 1/(1+((b/(1-b))^(1-vega))*)((1/(1-exp(-1/beta)))^((1-vega)/vega)));
a2 = 1/(exp(1/beta)-1);
neta = 1.0;
epsilon = 6.0;
```
Dynare (2)

li1 = 0.5;
li2 = 3.0;
li3 = 1.5;
li4 = 1.5;
rhoa = 0.75;
rhop = 0.75;
rhoi = 0.25;
rhom = 0.75;
pb = 2;
Write your log linearized DSGE model’s equations here

Example:

\[ r = (1-li1)*(li2*(pi-pb) + li3*(y-yf)) + li1*r(-1) + ei; \]
\[ at = rhoa * at(-1) + ua; \]
\[ ep = rhop * ep(-1) + up; \]
\[ ei = rhoi * ei(-1) + ui; \]
\[ em = rhom * em(-1) + um; \]
end;

steady;
check;
shocks;
var ua = 0.05;
var ui = 0.05;
var up = 0.05;
var um = 0.05;
end;

estimated_params;
   alpha, beta_pdf, 0.33, 0.05;
teta, beta_pdf, 0.66, 0.05;
vega, normal_pdf, 1.25, 0.05;
sigma, normal_pdf, 2.0, 0.5;
b, beta_pdf, 0.25, 0.1;
### Dynare (5)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>pb</code></td>
<td>normal_pdf</td>
<td>2.0</td>
<td>0.1</td>
</tr>
<tr>
<td><code>neta</code></td>
<td>normal_pdf</td>
<td>1.0</td>
<td>0.1</td>
</tr>
<tr>
<td><code>epsilon</code></td>
<td>normal_pdf</td>
<td>6.0</td>
<td>0.1</td>
</tr>
<tr>
<td><code>li1</code></td>
<td>beta_pdf</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td><code>li2</code></td>
<td>normal_pdf</td>
<td>3.0</td>
<td>0.5</td>
</tr>
<tr>
<td><code>li3</code></td>
<td>normal_pdf</td>
<td>1.5</td>
<td>0.5</td>
</tr>
<tr>
<td><code>li4</code></td>
<td>normal_pdf</td>
<td>1.5</td>
<td>0.5</td>
</tr>
<tr>
<td><code>rhoa</code></td>
<td>beta_pdf</td>
<td>0.75</td>
<td>0.1</td>
</tr>
<tr>
<td><code>rhop</code></td>
<td>beta_pdf</td>
<td>0.75</td>
<td>0.1</td>
</tr>
<tr>
<td><code>rhoi</code></td>
<td>beta_pdf</td>
<td>0.25</td>
<td>0.1</td>
</tr>
<tr>
<td><code>rhom</code></td>
<td>beta_pdf</td>
<td>0.75</td>
<td>0.1</td>
</tr>
</tbody>
</table>
Dynare (6)

```
stderr ua, inv_gamma_pdf, 0.05, 2;
stderr ui, inv_gamma_pdf, 0.05, 2;
stderr up, inv_gamma_pdf, 0.05, 2;
stderr um, inv_gamma_pdf, 0.05, 2;
end;
varobs pi y mp r;
estimation(nograph,order=1,datafile=databdet117m3pc,mode_compute=4,
forecast=12,mh_jscale=0.5,mh_replic=50000,mh_nbblocks=10) pi y mp r yf mpf;
shock_decomposition;
stoch_simul(nograph,order=2,irf=40,conditional_variance_decomposition=[1:60]);
```
## Bayesian estimation of structural parameters (1)

<table>
<thead>
<tr>
<th>Priors</th>
<th>Posterior $\sigma = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Law</td>
<td>Mean</td>
</tr>
<tr>
<td>$\alpha$ beta</td>
<td>0.33</td>
</tr>
<tr>
<td>$\theta$ beta</td>
<td>0.66</td>
</tr>
<tr>
<td>$\nu$ normal</td>
<td>1.25</td>
</tr>
<tr>
<td>$\sigma$ normal</td>
<td>2.00</td>
</tr>
<tr>
<td>$b$ beta</td>
<td>0.25</td>
</tr>
<tr>
<td>$\eta$ normal</td>
<td>1.00</td>
</tr>
<tr>
<td>$\varepsilon$ normal</td>
<td>6.00</td>
</tr>
<tr>
<td>$\lambda_i$ beta</td>
<td>0.50</td>
</tr>
<tr>
<td>$\lambda_\pi$ normal</td>
<td>3.00</td>
</tr>
<tr>
<td>$\lambda_x$ normal</td>
<td>1.50</td>
</tr>
<tr>
<td>$\lambda_m$ normal</td>
<td>1.50</td>
</tr>
<tr>
<td>$\pi_c$ normal</td>
<td>2.00</td>
</tr>
</tbody>
</table>
### Bayesian estimation of structural parameters (2)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Prior</th>
<th>Posterior (σ estimated)</th>
<th>Posterior (σ = 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_a$</td>
<td>beta</td>
<td>0.75</td>
<td>0.992</td>
<td>0.994</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>beta</td>
<td>0.75</td>
<td>0.973</td>
<td>0.972</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>beta</td>
<td>0.50</td>
<td>0.460</td>
<td>0.560</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>beta</td>
<td>0.75</td>
<td>0.971</td>
<td>0.984</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>invgamma</td>
<td>0.02</td>
<td>0.013</td>
<td>0.019</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>invgamma</td>
<td>0.02</td>
<td>0.018</td>
<td>0.012</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>invgamma</td>
<td>0.02</td>
<td>0.004</td>
<td>0.004</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>invgamma</td>
<td>0.02</td>
<td>0.026</td>
<td>0.027</td>
</tr>
</tbody>
</table>
Validation tools

- Distribution of the estimated shocks: stochastically around zero.
- Stable convergence of the within variance to the between and within variance, over the three first moments, of the multivariate Metropolis–Hastings convergence diagnosis.
- Simple look at the informativeness of the data by comparing prior and posterior distributions.
- Sensitivity analysis.
- Economic-relevant simulations: impulse response functions and variance decompositions.
Interval

m2

m3
First period variance decomposition (percent)

<table>
<thead>
<tr>
<th></th>
<th>estimated $\sigma$</th>
<th>$\sigma = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\epsilon_p^t$</td>
<td>$\epsilon_i^t$</td>
</tr>
<tr>
<td>$\hat{y}_t$</td>
<td>2.16</td>
<td>31.17</td>
</tr>
<tr>
<td>$\hat{\pi}_t$</td>
<td>77.72</td>
<td>22.16</td>
</tr>
<tr>
<td>$\hat{i}_t$</td>
<td>16.35</td>
<td>83.44</td>
</tr>
<tr>
<td>$\hat{mp}_t$</td>
<td>1.28</td>
<td>13.76</td>
</tr>
<tr>
<td>$\hat{y}_t^f$</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$\hat{mp}_t^f$</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
### Unconditional variance decomposition (percent)

<table>
<thead>
<tr>
<th></th>
<th>estimated $\sigma$</th>
<th>$\sigma = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon^p_t$</td>
<td>$\varepsilon^i_t$</td>
</tr>
<tr>
<td>$\hat{y}_t$</td>
<td>1.65</td>
<td>1.09</td>
</tr>
<tr>
<td>$\hat{\pi}_t$</td>
<td>97.66</td>
<td>2.14</td>
</tr>
<tr>
<td>$\hat{i}_t$</td>
<td>78.53</td>
<td>19.64</td>
</tr>
<tr>
<td>$\hat{m_p}_t$</td>
<td>1.85</td>
<td>0.91</td>
</tr>
<tr>
<td>$\hat{y}_t^f$</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$\hat{m_p}_t^f$</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Conditional variance decomposition of output
## Alternative Taylor rules (1)

<table>
<thead>
<tr>
<th></th>
<th>$\tilde{M}_{t,0}$</th>
<th>$\tilde{M}_{t,1}$</th>
<th>$\tilde{M}_{t,2}$</th>
<th>$\tilde{M}_{t,3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_i$</td>
<td>0.527</td>
<td>0.573</td>
<td>0.561</td>
<td>0.547</td>
</tr>
<tr>
<td>$(1 - \lambda_i) \lambda_{\pi}$</td>
<td>1.594</td>
<td>1.491</td>
<td>1.463</td>
<td>1.537</td>
</tr>
<tr>
<td>$(1 - \lambda_i) \lambda_{x}$</td>
<td>1.066</td>
<td>0.799</td>
<td>1.018</td>
<td>1.042</td>
</tr>
<tr>
<td>$(1 - \lambda_i) \lambda_{m}$</td>
<td>0.431</td>
<td>0.136*</td>
<td>0.084*</td>
<td></td>
</tr>
<tr>
<td>$ST_m^Y$ (%)</td>
<td>7.05</td>
<td>7.50</td>
<td>2.23</td>
<td>3.66</td>
</tr>
<tr>
<td>$LT_m^Y$ (%)</td>
<td>2.75</td>
<td>3.07</td>
<td>2.24</td>
<td>2.36</td>
</tr>
<tr>
<td>LMD</td>
<td>-629.8</td>
<td><strong>-618.2</strong></td>
<td>-634.9</td>
<td>-635.3</td>
</tr>
</tbody>
</table>

*estimations are not significant in terms of student tests ($t < 1.645$)
Alternative Taylor rules (2)

\[ \sigma = 4 \]

<table>
<thead>
<tr>
<th></th>
<th>( \tilde{M}_{t,0} )</th>
<th>( \tilde{M}_{t,1} )</th>
<th>( \tilde{M}_{t,2} )</th>
<th>( \tilde{M}_{t,3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_i )</td>
<td>0.545</td>
<td>0.614</td>
<td>0.546</td>
<td>0.547</td>
</tr>
<tr>
<td>( (1 - \lambda_i) \lambda_\pi )</td>
<td>1.579</td>
<td>1.345</td>
<td>1.585</td>
<td>1.575</td>
</tr>
<tr>
<td>( (1 - \lambda_i) \lambda_x )</td>
<td>1.034</td>
<td>0.741</td>
<td>1.038</td>
<td>1.039</td>
</tr>
<tr>
<td>( (1 - \lambda_i) \lambda_m )</td>
<td>0.371</td>
<td>-0.012*</td>
<td>-0.018*</td>
<td>-0.018*</td>
</tr>
<tr>
<td>( ST_m^\gamma ) (%)</td>
<td>22.61</td>
<td>22.38</td>
<td>23.20</td>
<td>23.28</td>
</tr>
<tr>
<td>( LT_m^\gamma ) (%)</td>
<td>9.56</td>
<td>10.38</td>
<td>9.29</td>
<td>9.15</td>
</tr>
<tr>
<td>LMD</td>
<td>-639.8</td>
<td>-626.5</td>
<td>-646.1</td>
<td>-646.1</td>
</tr>
</tbody>
</table>

*estimations are not significant in terms of student tests (t<1.645)
Role of Money

- Whatever the formulation of the Taylor rule, the estimated parameters of the whole model are quite similar.
- The impact of a money shock on output are also rather similar whatever the Taylor rule.
- The weight of the money shock on output dynamics, $\kappa_{sm}$, and on flexible-price output, $\nu^y_{sm}$, increases with risk aversion.
- **The higher the risk aversion, the higher the role of money on output.**
- The explicit money variable does not appear to have a notable direct role in explaining inflation variability.
- One may infer that by changing economic agents’ perception of risks, the last financial crisis may have increased the role of money in the transmission mechanisms and in output changes.
Monetary Policy

- The central bank strives for financial stability in crisis periods. The smoothing parameter in the Taylor rule equation, $\lambda_i$, increases with risk aversion.

- **The higher the risk aversion, the stronger the smoothing of the interest rate.** This reflects probably the central bankers’ objective not to agitate markets.

- The introduction, or not, of a money variable in the ECB monetary policy reaction function does not really appear to change significantly the impact of money on output and inflation dynamics.

- Our results suggest that a nominal or real money growth variable does not improve the estimated ECB monetary policy rule. Yet, a **real money gap** variable significantly improves the estimated Taylor rule.
PROGRESS AND INCENTIVES IN MACROECONOMICS

I'm working on a path-breaking new paper on X, which was a major contributor to the Great Financial Crisis. Better understanding of X will help us prevent the next crisis and save millions of people from starvation! I'm really excited about it!

Surely you're also including D, S, GE and, of course, RE in your model? Actually, when I mix all these ingredients together with X, it takes my computer 1000 years to solve the model. So I stopped by to ask for your advice on how to simplify the problem.

Well, it's probably best if you leave out X from your model. You'll still have a solid paper using just D, S and GE.

I'll write you a strong recommendation and you'll land an excellent job at a top university. Your peers will envy you.

You can still work on X once you've got your job... Or maybe better, after you've got tenure... or in retirement.
The foundations

- **Intertemporal optimization.** In deciding how much to consume, consumers think not only about current income but also about future income; that in deciding how much to invest, firms think not only about current but also future profitability; and so on.

- **Nominal rigidities.** Macroeconomics would be easier and more elegant without nominal rigidities. Unfortunately, they do exist. The price level is a sum of very many individual prices, each of them set for some period of time; this fact drastically changes price-level dynamics.

- **Imperfect competition.** That goods, labor, and credit markets are not perfectly competitive is nearly self-evident. That these imperfections are central to macroeconomic is more controversial, but not much.
Some answers in Oxford Review of Economic Policy


- Blanchard : **On the future of macroeconomic models.**
- Lindé : **DSGE models: still useful in policy analysis ?**
- Reis : **Is something really wrong with macroeconomics?**
- Ghironi : **Macro needs micro.**
Some paths in macroeconomics

- Behavioral
  - Bounded rationality (myopia), time inconsistency (present bias), etc.
- Expectation formation
  - Learning: adaptative learning, etc.
- Heterogenous agents
  - HANK, incomplete markets etc.
- Financial frictions
  - LTV constraints, asset pricing, etc.
- Additional sectors
  - Banking, housing, fiscal authority, etc.
- Additional economies
  - Open economy, SOE, etc.
Thank you for your attention

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▶ Website: JonathanBenchimol.com
Does risk aversion impacted output and real money balance dynamics differently?
Baseline model (presented above) + risk aversion shock.

Periods:
- **P1**: from 1971 Q1 to 1991 Q1.
- **P2**: from 1976 Q1 to 1996 Q1.
- **P3**: from 1981 Q1 to 2001 Q1.
- **P4**: from 1986 Q1 to 2006 Q1.
- **P5**: from 1991 Q1 to 2011 Q1.
Figure: Bayesian estimation of parameters over the selected periods
Figure: Variance decomposition over the selected periods
Figure: Impulse response function with respect to a price-markup shock
**Figure:** Impulse response function with respect to a technology shock
**Figure:** Impulse response function with respect to a technology shock
Figure: Impulse response function with respect to a risk aversion shock